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RADIATION PRESSURE IN ACOUSTIC WAVE CALCULATIONS OF EARLY-TYPE STARS

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ABSTRACT. A new method to treat radiation pressure in spectral lines has been developed which avoids the Sobolev-approximation. This method has been used for time-dependent acoustic wave calculations in early-type stars.

1. INTRODUCTION

For early-type stars spectral lines constitute the most important contribution to the radiation pressure force. In the treatments of Castor (1974) as well as Castor et al. (1975) the Sobolev-approximation has been used which however breaks down when acoustic waves in regions of small wind velocity are considered. For these cases a new method has been developed to compute the time-dependent radiation pressure force.

2. METHOD TO COMPUTE THE RADIATION PRESSURE

To take into account the effects of an intense radiation field in the time-dependent hydrodynamic equations (for a review of time-dependent atmospheric wave calculations in stellar atmospheres see Ulmschneider and Muchmore 1986) the energy equation does not need to be modified and reads (Wolf 1985, 1986b)

$$\Phi_R = -4\pi\kappa_R'(J-B) \quad , \tag{1}$$

where Φ_R is the net radiative cooling rate, J the frequency integrated mean radiative intensity, B the Planck function and κ_R' the Rosseland opacity without scattering. In the Euler equation a radiative force term

$$\rho g_R = \frac{1}{c}\int_0^\infty \kappa_\nu F_\nu d\nu \simeq \frac{1}{c}(\kappa_R+\kappa_L)\ F \ , \tag{2}$$

has to be added where ρ is the density, F_ν is the monochromatic and F

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the frequency integrated radiative flux. Here κ_ν is the monochromatic opacity with scattering, κ_R the Rosseland opacity (continuum contribution) and κ_L is a flux weighted mean opacity (line contribution). To make the problem tractable Wolf (1985) assumed LTE, a two beam approximation with no ingoing radiation field and that the line contribution arises at heights, where the continuum optical depth has fallen below unity and the lines can be considered generated by pure absorption in a fixed incident continuum radiation field. We then find

$$\kappa_L = \sum_{i,k} \chi_L(x) \int_0^\infty \varphi_\nu(x) \frac{F_\nu}{F} \exp(-\tau_\nu{}^L(x))\, d\nu \quad , \tag{3}$$

where F_ν and F are photospheric fluxes. The contribution of element i and ionization stage k, assuming an atom with two levels L,U (Mihalas 1978, p.336) is

$$\chi_L = \frac{\pi e^2}{m_e c} f_{LU}\, n_a\, X_i\, \frac{n_{ik}}{n_i} \frac{1-\exp(-h\nu_{LU}/kT)}{1+g_U \exp(-h\nu_{LU}/kT)/g_L} \quad . \tag{4}$$

Here n_a is the number density of heavy particles, X_i the fractional abundance by number, f_{LU} the oscillator strength, and n_i, n_{ik} are number densities computed in LTE. The flux F_ν is attenuated over the optical depth $\tau_\nu{}^L$ from the photospheric height x_p of the star to height x along a ray inclined by angle cosine $3^{-1/2}$:

$$\tau_\nu{}^L(x) = \int_{x_p}^{x} \chi_L(x')\varphi_\nu(x')\, 3^{1/2} dx' \quad , \tag{5}$$

As the flux F_ν relevant for the radiative force at height x (using the Barbier-Eddington approximation) emerges from a distance $\Delta\tau=2/3$ closer to the star, the integration in Eq. (5) is carried out only to height $x_{2/3}$ (c.f. Wolf 1985). Following Castor et al. (1975) we approximate the ratio F_ν/F by the ratio B_ν/B of the Planck functions which can be taken out of the integral of Eq. (3). Assuming a rectangular line profile for which the central depth ϕ_L is chosen to match the relevant Voigt profile we have a line width $2\Delta\nu_L=1/\phi_L$. Fig. 1 shows the line profile band traced out by an acoustic wave motion in the (x,ν) plane. The fluid element at height x sees the radiative flux F_ν emerging from height $x_{2/3}$. The flux F_ν is present only if the frequency is outside the shaddow zone $\nu_R \leq \nu \leq \nu_B$ because it is assumed that any flux inside the shaddow zone has been absorbed. The limits ν_B and ν_R are given by

$$\begin{aligned}(\nu_B-\nu_{LU})/\Delta\nu_D &= \Delta\nu_L/\Delta\nu_D + 3^{-1/2}\, u_{MAX}/v_{th} \quad ,\\ (\nu_R-\nu_{LU})/\Delta\nu_D &= -\Delta\nu_L/\Delta\nu_D + 3^{-1/2}\, u_{MIN}/v_{th} \quad ,\end{aligned} \tag{6}$$

where ν_{LU} is the frequency of the line center, $\Delta\nu_D$ the Doppler width and v_{th} the thermal speed of the line forming ion. For every x $u_{MAX}(x)$ is th

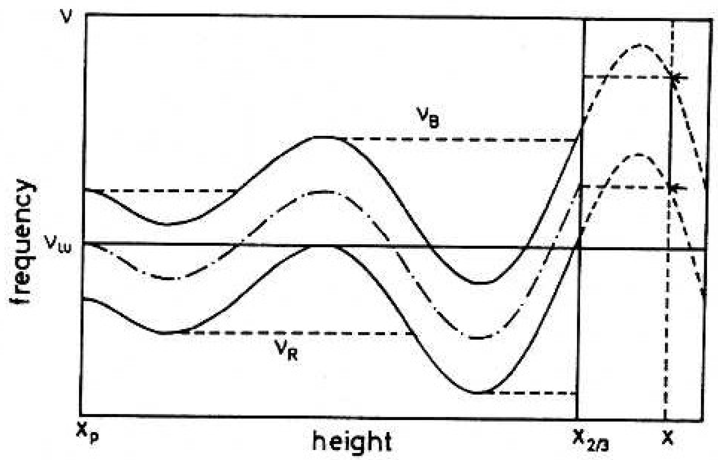

Figure 1. Height-frequency diagram for an acoustic wave

maximum value of the velocity of the wave in the height interval x_p to x, and $u_{MIN}(x)$ the corresponding minimum of the velocity in the same height interval. Assuming that $\tau_\nu^L(x)$ of Eq. (5) can be approximated by ∞ if $u_{MIN} \le u(x) \le u_{MAX}$ and by 0 else, then from Eq. (3) using

$$f(u,x) = \begin{cases} 2\Delta\nu_L/\Delta\nu_D & , \text{ if } u > u_{MAX}+u_{th} \text{ or } u < u_{MIN}-u_{th} \\ 0 & , \text{ if } u_{MIN} \le u \le u_{MAX} \\ (u-u_{MAX})/v_{th} & , \text{ if } u_{MAX} \le u \le u_{MAX}+u_{th} \\ (u_{MIN}-u)/v_{th} & , \text{ if } u_{MIN}-u_{th} \le u \le u_{MIN} \end{cases} \quad , \tag{7}$$

$$u_{th} = 12^{1/2} \, v_{th} \, \Delta\nu_L/\Delta\nu_D \quad , \tag{8}$$

we obtain

$$\kappa_L = \sum_{i,k} \chi_L(x) \, B_\nu/B \, \phi_L \, \Delta\nu_D \, f(u,x) \quad . \tag{9}$$

For the line contributions there are three different effects: strong resonance lines with little flux in the line core can be Doppler shifted into the continuum radiation field, optically thin lines contribute because there are so many of them, new ions which see an undiluted radiation field can be created by ionization in the hot compression region behind the shocks. The thin line contribution can be evaluated by using the first case in Eq. (7), by which $\phi_L \Delta\nu_D f(u,x) = 1$ in Eq. (9). This contribution thus becomes independent of u and is a function of temperature T and pressure p only. The same is the case for the new ions which see an undiluted flux. Using some mean thermal speed v_{th} in Eq.'s (7) and (9), the velocity dependent contribution of the resonance lines can likewise be evaluated as function of T and p. We thus find

$$\rho g_R = F\{\kappa_R(T,p)+(1-w)\kappa_1(T,p)f(u,x)+w\kappa_2(T,p)+(1-w)\sum_{i=3}^{10}\kappa_i(T,p)\}/c \quad , \quad (10)$$

where κ_R is the Rosseland opacity with scattering and κ_1 to κ_{10} are line opacities. w is a weighting function which decreases with height. The Rosseland opacity describes the continuum contribution. κ_1 is the contribution from the resonance lines, κ_2 from the optically thin lines and κ_3 to κ_{10} from the strong lines of new ions. f(u,x) describes the Doppler shift of resonance lines into the undiluted continuum. The contribution from the thin lines and new ions does not depend on velocity. Consider a grid point at some height x with temperature T and pressure p, while at the next grid point closer to the star one has T' and p'. There are eight possibilities of T, p being greater, equal or less than T', p'. If between T', p' and T, p a new ion appears its opacity contribution is listed in the tables κ_3 to κ_{10}. The advantage of this approach is that the opacities κ_1 to κ_{10} can be pretabulated and summed over a large number of lines. Wolf(1985, 1986b) took up to 650 lines and showed that the important contributions came from the elements H, He, C, N, O, Na, and Fe.

3. RESULTS AND CONCLUSIONS

The present method avoids the Sobolev approximation, and thus is well suited for wave calculations. A large number of wave computations for stars of spectral types O3V to A0V, B0III to B5III and O5I to B5I were made (Wolf 1985, 1986d) using initial acoustic fluxes corresponding to a Mach number 0.01 and periods of 0.4 times the photospheric cut-off period. The calculations were started at a continuum optical depth of between 3 and 10^{-3}. It was found that there is a radiative damping zone up to an optical depth of typically 10^{-5} where the waves are essentially isothermal with a relative temperature fluctuation $\Delta T/T$ of around 10^{-7}. This also applied to the shocks formed in this zone. After passing the upper limit of the damping zone the waves by radiative acceleration rapidly grew to strong shocks with postshock temperatures of the order 3 10^6 K. The generated stellar wind could not be followed because of the limited width of the atmospheric slab which extended to continuum optical depths of around 10^{-9}. The magnitude of the radiative amplification in the various stars was discussed in detail. Radiation pressure in lines was found to be the dominant contribution.

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