The Role of Fine-Scale Magnetic Fields on the Structure of the Solar Atmosphere

Proceedings of the Inaugural Workshop
and Round Table Discussion for the
D-E-S Telescope Installations on the Canary Islands
La Laguna, Tenerife, Spain 6–12 October 1986

Editors
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Sponsored jointly by

(D) Deutsche Forschungsgemeinschaft, Bonn

(E) Instituto de Astrofisica de Canarias, La Laguna

(S) The Royal Swedish Academy of Sciences, Stockholm

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ADIABATIC LONGITUDINAL-TRANSVERSE MAGNETOHYDRODYNAMIC TUBE WAVES

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Abstract. We compute the propagation of adiabatic magnetohydrodynamic waves along thin flux tubes in the solar atmosphere. The time-dependent development and amplitude growth of the waves, excited at the foot of the photosphere as pure transverse waves, was followed into the low chromosphere. Strong mode-coupling to longitudinal waves was found. The swaying of the tube which increases with height resulted in a lifting of the entire tube mass which we attribute to centrifugal forces. This lifting resulted in adiabatic cooling of the tube.

1. Introduction

As discussed by Spruit (1982) thin magnetic flux tubes allow three types of tube wave modes: longitudinal, transverse and torsional waves. Despite the fact that these waves are probably all excited very efficiently it is interesting to investigate their non-linear coupling especially as this may have great importance for both chromospheric and coronal heating (c.f. Ulmschneider 1987). Hollweg et al. (1982) have computed the time-development of coupled longitudinal and torsional waves. They showed that from a purely torsional excitation longitudinal waves of considerable energy developed. In the present paper we want to report similar time-dependent work on coupled longitudinal and transverse waves.

2. Method

Following Spruit (1981) we assume a vertically oriented thin magnetic flux tube (c.f. Fig. 1) in which a mass element of width da at time t=0 can be uniquely identified by the Lagrange height, a, measured in the outward direction from the level where the continuum optical depth outside the tube is one. At a=0 the tube has a diameter of 100 km and a magnetic field strength of B_o=1500 G (c.f. Tab. 1). The tube is assumed

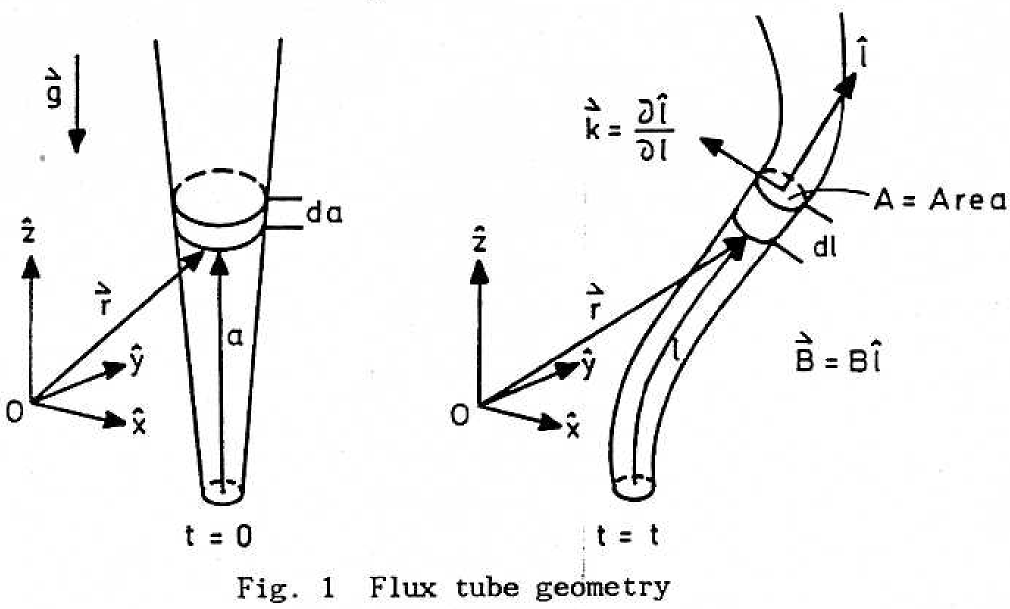

Fig. 1 Flux tube geometry

to spread with height almost exponentially in pressure balance in an external unmagnetized radiative equilibrium atmosphere similarly as discussed by Herbold et al. (1985). At some later time t (cf. Fig. 1) the tube is assumed to have kinks and the cross-sectional area A may be changed. The mass element has now the width dl and is displaced to a

a (km)	d_o (km)	B_o (G)	p_e $\left[\frac{dyn}{cm^2}\right]$	p_o $\left[\frac{dyn}{cm^2}\right]$	T (K)	ρ_e $\left[\frac{g}{cm^3}\right]$	ρ_o $\left[\frac{g}{cm^3}\right]$	c $\left[\frac{km}{s}\right]$	c_A $\left[\frac{km}{s}\right]$	c_T $\left[\frac{km}{s}\right]$	c_F $\left[\frac{km}{s}\right]$
0	100	1500	1.4(5)	4.7(4)	6049	3.6(- 7)	1.2(- 7)	8.0	12.2	6.7	6.1
832	637	37	8.3(1)	2.8(1)	4705	2.8(-10)	9.4(-11)	7.1	10.7	5.9	5.4

Tab.1. Diameter d_o, magnetic field strength B_o, external p_e and internal gas pressure p_o, temperature T, external ρ_e and internal density ρ_o, soundspeed c, Alfvenspeed c_A, tube speed c_T, and speed c_F as function of the (Lagrange) height a in the initial tube model.

position described by the arc length l(a,t). The unit vector in direction l is defined by:

$$\hat{l} \equiv \left[\frac{\partial\vec{r}}{\partial l}\right]_t = \frac{1}{l_a}\left[\frac{\partial\vec{r}}{\partial a}\right]_t = (l_x, l_y, l_z), \quad \text{with} \quad l_x^2 + l_y^2 + l_z^2 = 1, \tag{1}$$

and l_a defined below. A curvature vector is defined by

$$\vec{K} \equiv \left[\frac{\partial\hat{l}}{\partial l}\right]_t = \frac{1}{l_a}\left[\frac{\partial\hat{l}}{\partial a}\right]_t \perp \hat{l} \quad . \tag{2}$$

The position of the mass element may also be described by the radius vector $\vec{r}$ from an Eulerian coordinate system (cf. Fig. 1). The magnetic field is assumed to be exclusively in the $\hat{l}$ direction. The conservation of mass requires

$$\rho(a,t)\ A(a,t)\ dl = \rho_o(a)\ A_o(a)\ da \quad , \tag{3}$$

where ρ is the density and the subscript o denotes values at time t=0. Eq. (3) defines a scale factor

$$l_a \equiv \left[\frac{\partial l}{\partial a}\right]_t = \frac{\rho_o\ A_o}{\rho\ A} \quad . \tag{4}$$

We first consider Eulers equation:

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right] = -\nabla p - \frac{1}{4\pi}\vec{B}\times(\nabla\times\vec{B}) + \rho\vec{g} \quad . \tag{5}$$

For the transverse component of this Eq. one has (Spruit 1981):

$$\rho\left[\left[\frac{\partial\vec{v}}{\partial t}\right]_a - \left[\hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial t}\right]_a\right]\hat{l}\right] = -\left[\hat{l}\times\nabla(p+\frac{B^2}{8\pi})\right]\times\hat{l} + \frac{B^2}{4\pi}\vec{K} + \rho(\hat{l}\times\vec{g})\times\hat{l} \quad . \tag{6}$$

As the transverse wave moves also mass outside the tube we replace ρ in the inertia term by $\rho+\rho_e$. Using $p+B^2/8\pi = p_e(z)$ one finds

$$\left[\frac{\partial\vec{v}}{\partial t}\right]_a - \left[\hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial t}\right]_a\right]\hat{l} = \frac{\rho c_A^2}{\rho+\rho_e}\frac{1}{l_a}\left[\frac{\partial\hat{l}}{\partial a}\right]_t + \frac{\rho-\rho_e}{\rho+\rho_e}(\vec{g}+g l_z\hat{l}) \quad , \tag{7}$$

with c_A = Alfvenspeed. For the longitudinal component one has:

$$\rho\hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial t}\right]_a = -\frac{1}{l_a}\left[\frac{\partial p}{\partial a}\right]_t - \rho g l_z \quad . \tag{8}$$

We now consider the induction and continuity equations:

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}) = \vec{v}(\nabla\cdot\vec{B}) + (\vec{B}\cdot\nabla)\vec{v} - \vec{B}(\nabla\cdot\vec{v}) - (\vec{v}\cdot\nabla)\vec{B} \quad , \tag{9}$$

$$\frac{\partial\rho}{\partial t} + (\vec{v}\cdot\nabla)\rho + \rho(\nabla\cdot\vec{v}) = 0 \quad . \tag{10}$$

Using these Eqs. we find for the transverse component:

$$\left[\frac{\partial\hat{l}}{\partial t}\right]_a = -\frac{1}{l_a}\left[\left[\frac{\partial\vec{v}}{\partial a}\right]_t - \left[\hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial a}\right]_t\right]\hat{l}\right] \quad . \tag{11}$$

and for the longitudinal component:

$$\left[\frac{\partial B}{\partial t}\right]_a = \frac{B}{\rho}\left[\frac{\partial\rho}{\partial t}\right]_a + \frac{B}{l_a}\hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial a}\right]_t \quad . \tag{12}$$

From the time derivative of $B^2/8\pi = p_e - p$ we have

$$\frac{\rho c_A^2}{B}\left[\frac{\partial B}{\partial t}\right]_a = \left[\frac{\partial}{\partial t}(p_e - p)\right]_a = v_z\frac{dp_e}{dz} - \left[\frac{\partial p}{\partial t}\right]_a \text{ , and with Eq. (12):}$$

$$\frac{\hat{l}}{l_a}\cdot\left[\frac{\partial\vec{v}}{\partial a}\right]_t = -\frac{1}{\rho c_A^2}\left[\frac{\partial p}{\partial t}\right]_a - \frac{1}{\rho}\left[\frac{\partial\rho}{\partial t}\right]_a + \frac{v_z}{\rho c_A^2}\frac{dp_e}{dz} \quad . \tag{13}$$

Eliminating ρ and p in favour of the sound speed c and the entropy S we now have the following 5 equations, Eulers equation:

$$\parallel \quad \hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial t}\right]_a + \frac{1}{l_a}\left[\frac{2c}{\gamma-1}\left[\frac{\partial c}{\partial a}\right]_t - \frac{c^2\mu}{\gamma R}\left[\frac{\partial S}{\partial a}\right]_t\right] + g l_z = 0 \quad , \tag{14}$$

$$\perp \quad \left[\frac{\partial\vec{v}}{\partial t}\right]_a - \hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial t}\right]_a - \frac{\rho}{\rho+\rho_e}\frac{c_A^2}{l_a}\left[\frac{\partial\hat{l}}{\partial a}\right]_t - \frac{\rho-\rho_e}{\rho+\rho_e}(\vec{g} + g l_z\hat{l}) = 0. \tag{15}$$

Induction and continuity equations:

$$\parallel \quad \frac{\hat{l}}{l_a}\cdot\left[\frac{\partial\vec{v}}{\partial a}\right]_t + \frac{2c}{\gamma-1}\frac{1}{c_T^2}\left[\frac{\partial c}{\partial t}\right]_a - \frac{\mu}{\gamma R}\left[\frac{c^2}{c_A^2} + \gamma\right]\left[\frac{\partial S}{\partial t}\right]_a - \frac{v_z}{\rho c_A^2}\frac{dp_e}{dz} = 0, \tag{16}$$

$$\perp \quad \left[\frac{\partial\vec{v}}{\partial a}\right]_t - \hat{l}\cdot\left[\frac{\partial\vec{v}}{\partial a}\right]_t - l_a\left[\frac{\partial\hat{l}}{\partial t}\right]_a = 0 \quad . \tag{17}$$

where $c_T = \left[c^2c_A^2 / (c^2 + c_A^2)\right]^{1/2}$ is the tube speed.

Entropy conservation equation:

$$\left[\frac{\partial S}{\partial t}\right]_a = \left.\frac{dS}{dt}\right|_{rad} \quad . \tag{18}$$

Going over to characteristic form these equations read:

$$\hat{l}\cdot d\vec{v} \pm \frac{2}{\gamma-1}\frac{c}{c_T}dc \mp \frac{\mu c^2}{\gamma R c_T}dS \mp \left[\frac{\mu c_T}{\gamma R}(\gamma-1)\left.\frac{dS}{dt}\right|_{rad} + \frac{v_z c_T}{\rho c_A^2}\frac{dp_e}{dz} - g l_z\right]dt = 0, \tag{19}$$

along the $C_2^{\pm}$ characteristics given by $\frac{da}{dt} = \pm\frac{c_T}{l_a}$. (20)

where Eqs. (19), (20) are those of Herbold et al. (1985) for $l_z = 1$, and

$$(1-l_x^2)dv_x - l_x l_y dv_y - l_x l_z dv_z \mp c_F dl_x - \frac{\rho-\rho_e}{\rho+\rho_e} g l_x l_z dt = 0 \quad , \tag{21}$$

$$(1-l_y^2)dv_y - l_x l_y dv_x - l_y l_z dv_z \mp c_F dl_y - \frac{\rho-\rho_e}{\rho+\rho_e} g l_y l_z dt = 0 \quad , \tag{22}$$

along the $C_1^{\pm}$ characteristics given by $\frac{da}{dt} = \pm\frac{c_F}{l_a} \equiv \pm\sqrt{\rho/(\rho+\rho_e)}\,\frac{c_A}{l_a}$. (23)

3. Results

We have solved the timedependent equations described above for a case of an adiabatic wave and purely transverse shaking at the bottom of the tube assuming

$$v_{\perp x} = -v_o\sin(2\pi t/P), \qquad v_{\perp y} = 0, \qquad v_{\parallel} = 0, \tag{24}$$

with v_o=0.5 km/s and transmitting boundary conditions at the top.

Transmission was achieved assuming $v_{\parallel}$ = const along C_2^+ and $v_{\perp x}, v_{\perp y}$ =

const along C_1^+. The wave period was taken to be P=45s. As this shaking took place only in the x-direction the wave is confined to the x-z plane.

Fig. 2 shows a snapshot of the wave after 755 time steps. Note that the physical variables are shown here as function of the Lagrange height a. The curve labeled x shows the horizontal position of the center of

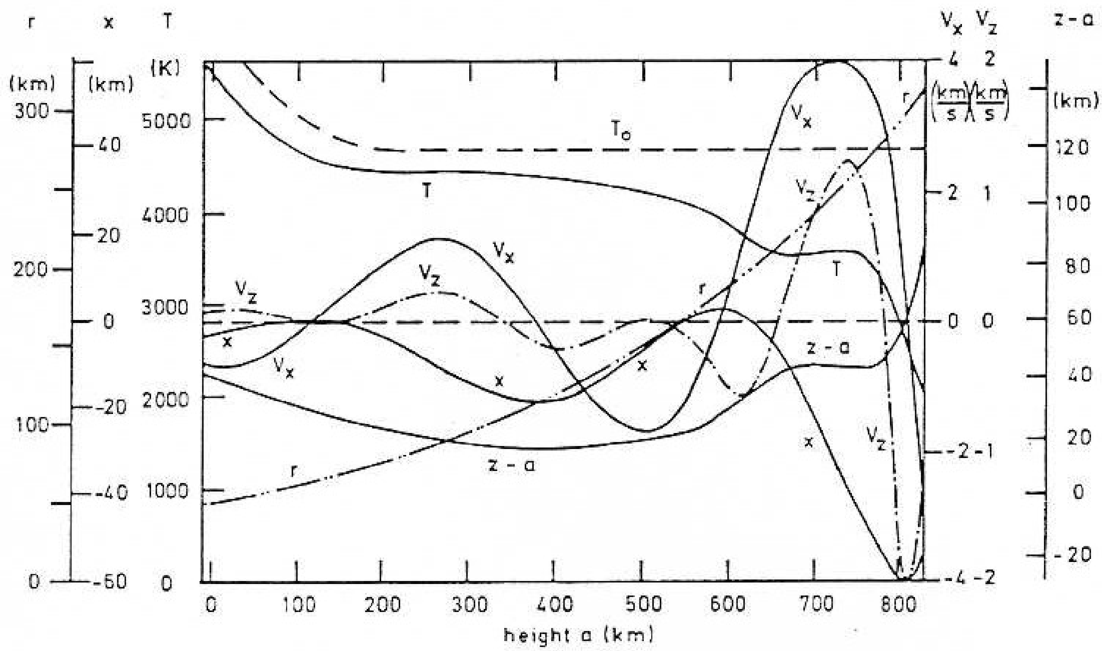

Fig. 2 Snapshot of the wave calculation at time t=1010.4 s, r=tube radius, other symbols are explained in the text.

the flux tube. At 800 km height the tube center is displaced by 60 km in the -x direction. This is about 10 percent of the tube diameter of 600 km at this height. The maxima and minima of the x curve corresponding to dx/da=0 coincide nicely with the nodes of the horizontal velocity v_x. The amplitude of v_x increases as a function of height due to flux conservation. With the propagation speed c_F roughly constant (Tab. 1) the amplitude of v_x grows roughly like $\rho^{-1/2}$. It is seen that v_x has a maximum of 4 km/s at 800 km height. At that height the x-curve has an inflection point where the gas elements in the tube, due to the motion of the wave in +z direction, are horizontally displaced with maximum speed in +x direction.

Fig. 2 shows that in addition to the transverse wave a longitudinal wave is generated where the vertical velocity v_z due to mode coupling shows a surprisingly large amplitude of 2 km/s, that is roughly one half the amplitude of v_x. Moreover it is seen in Fig. 2 that the longitudinal wave has twice the frequency of the transverse wave. This is understood if one decomposes the curvature force vector into vertical and horizontal components. In one wavelength of the x-curve the z-component of the curvature force vector changes sign four times while the x-component changes sign only two times.

Another surprising result of the present calculation is that the entire mass of the tube appears to be lifted relative to the initial position. Fig. 2 shows as function of height the distances z-a which are the height differences between the current (Eulerian) position of the gas elements and the (Lagrange) height (= the height at the start of the computation). It is seen that the bottom of the tube is lifted by about 40 km, the top by about 90 km, while everywhere else the tube is lifted by at least 20 km. The reason for this lifting appears to be the increase of the horizontal motion with height. As the bottom of the tube does not move much in the horizontal direction the large horizontal motion of the higher tube regions generates centrifugal forces, which, always outwardly directed, lead to the lifting of the tube. The result of this lifting is an adiabatic cooling of the entire tube as can be seen in Fig. 2 by comparing the current T and initial T_0 temperatures.

4. Conclusions

We found that purely tranverse excitation of magnetic flux tubes will lead to transverse waves with amplitudes strongly growing with height. In addition due to strong mode coupling, longitudinal waves were generated with amplitudes only a factor of two smaller than those of the transverse waves. The wave period of the longitudinal wave was one half that of the transverse wave. The horizontal swaying of the tube increases strongly with height and resulted in centrifugal forces which lead to a lifting and adiabatic cooling of the entire gas column in the flux tube. This part of our calculation would certainly be modified if radiative damping were taken into account. In addition the analogy to other free-boundary organ pipe-type wave motions suggests strong resonance effects i.e. dependences on the chosen wave period. These effects will be considered in our forthcoming work elsewhere.

Acknowledgement The authors gratefully acknowledge generous support by the Sonderforschungsbereich 132 and by the Deutsche Forschungsgemeinschaft.

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