Acoustic Heating

Peter Ulmschneider
Institut für Theoretische Astrophysik der Universität Heidelberg
Im Neuenheimer Feld 561, 6900 Heidelberg 1, Federal Republic of Germany.

Abstract. Acoustic shock waves are a viable and prevalent heating mechanism both in early- and in late-type stars. Acoustic heating appears to be a dominant mechanism for situations where magnetic fields are weak or absent, as locally for certain stellar surface regions, and globally for very slowly rotating stars with intrinsically weak magnetic fields, like the basal flux stars, the giants and supergiants. It also seems to be an important mechanism in F-stars, where the acoustic flux production is at a maximum. For more rapidly rotating stars, acoustic heating is a weak background effect. The limiting shock strength behaviour and radiation damping of acoustic waves are discussed. Both the directly observed solar acoustic wave flux and the empirical solar chromospheric cooling flux agree with the theoretical limiting shock wave flux. The heating by *pulsation-driven waves* is by hydrodynamic shocks and thus is a type of acoustic heating. In strong magnetic fields *slow-mode mhd waves* or *longitudinal tube waves* are essentially acoustic waves which also dissipate by hydrodynamic shocks. These waves are discussed with the magnetic heating mechanisms.

1. The acoustic heating theory

Only a few years after Edlén's (1941) discovery that the solar corona is a gas layer with a million degree temperature, a theoretical explanation for this hot layer, which attributes the heating to the dissipation of acoustic shock waves was offered by Biermann (1946, 1948) and by Schwarzschild (1948). This so called acoustic heating theory works as follows: In *late- type stars* (see Fig. 1) the turbulent convection zone beneath the stellar surface produces fluctuations in pressure and divergence of the Reynolds stress which act as sources for the generation of acoustic waves. These waves run down the steep density gradient of the outer stellar atmosphere, and, due to energy conservation, grow to large amplitude and form hydrodynamic shocks which heat the outer stellar layers. Direct viscous or thermal conductive heating by acoustic waves is many orders of magnitude too small. However, in the high chromosphere hydrogen ionization pumping by acoustic waves will be important (see Lindsey 1991, this volume). As all convection zones generate acoustic energy, *acoustic heating will be present in all late- type stars.*

The acoustic heating mechanism also works for *early-type stars* (see Fig. 1) where surface convection zones no longer exist. Here the intense radiation field amplifies small acoustic disturbances until strong acoustic shock waves develop. Time- dependent calculations of the radiative amplification of small acoustic disturbances into strong shocks have been performed by Wolf (1987) and Owocki et al. (1988).

Another type of acoustic heating is what could be called *pulsational heating* and occurs in Mira- or Semiregular Variables. Here global scale stellar pulsations and nonradial oscillations generate long period hydrodynamic shocks which heat the outer stellar atmosphere.

There is a third type of acoustic heating. In strong magnetic fields or flux tubes where the Alfvén speed is larger than the sound speed, slow-mode mhd waves or longitudinal tube waves

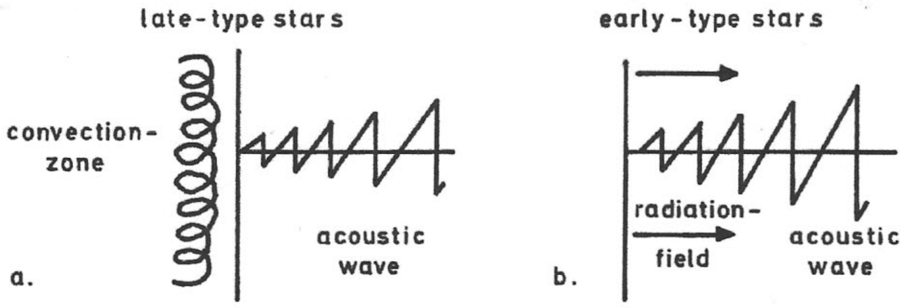

Figure 1. Panel a: Acoustic heating in late-type stars. The acoustic wave generation depends only on the three parameters which determine the convection zone: effective temperature T_{eff}, gravity g and mixing length parameter α. Panel b: Acoustic heating in early-type stars. Here the acoustic wave generation depends on the radiation field.

propagate along the field lines. These waves are essentially acoustic waves (see e.g. Herbold et al. 1985) and like acoustic waves dissipate by hydrodynamic shocks. As the propagation and the generation of these waves are intimately connected with the magnetic field they constitute a highly localized heating mechanism which is very different from the globally acting acoustic heating mechanism. For this reason it is customary to discuss these waves among the magnetoacoustic wave heating mechanisms (see Stein 1991, this volume). For recent reviews of acoustic heating see Kuperus et al. (1981), Narain and Ulmschneider (1990, henceforth called NU) and Ulmschneider (1986, 1990).

2. Weak shocks, limiting shock strength

Two basic properties are essential for the understanding of the behaviour of acoustic waves in stellar atmospheres: shock formation with the tendency to form sawtooth waves of limiting strength and radiation damping. Let us first discuss the limiting shock strength behaviour. For linear small amplitude sawtooth waves with pressure and velocity variations $p = p_0 + p_m - 2p_m t/P$, $v = v_m - 2v_m t/P$, where P is the wave period, t the time and subscript m indicating maximum amplitude the wave energy flux ($erg\ cm^{-2}\ s^{-1}$) is given by:

$$F_M = \frac{1}{P}\int_0^P (p - p_0)\ v\ dt = \frac{1}{3}\ p_m v_m \approx \frac{1}{12}\ \gamma\ p_0\ c_S\ \eta^2 \quad , \tag{1}$$

where p_0 is the unperturbed pressure, γ is the ratio of specific heats and where for weak shocks one has for the total pressure-, velocity-, temperature- and density jumps $2p_m \approx \gamma p_0 \eta$, $2v_m \approx c_S \eta$, $2T_m \approx (\gamma-1)T_0\eta$, $2\rho_m \approx \rho_0\eta$. Here the shock strength is defined as $\eta \equiv (\rho_2 - \rho_1)/\rho_1$, where ρ_1, ρ_2 are the densities in front and behind the shock (Ulmschneider 1970, Bray and Loughhead 1974). The shock dissipation rate ($erg\ cm^{-3}\ s^{-1}$) of the wave can be written

$$\epsilon_M = \frac{\rho T \Delta S}{P} = \frac{\rho_0\ c_S^2}{\gamma(\gamma-1)P}\ln\Big(\frac{p_2}{p_1}\Big(\frac{\rho_2}{\rho_1}\Big)^{-\gamma}\Big) \approx \frac{1}{12}\ \frac{\gamma(\gamma+1)}{P}\ p_0\ \eta^3 \quad , \tag{2}$$

where ΔS is the entropy jump per unit mass at the shock front. The approximate equality is only valid for weak shocks, where the entropy jump is small in third order of η. Let us assume a gravitational atmosphere and in analogy to ray optics that the quantity $F_M c_S^2$ is conserved. Differentiating with respect to height z and using Eq. (2) gives an equation for the shock strength

$$\frac{d\eta}{dz} = \frac{\eta}{2}\Big(\frac{\gamma g}{c_S^2} - \frac{3}{2c_S^2}\frac{dc_S^2}{dz} - \frac{(\gamma+1)\ \eta}{c_S P}\Big) \quad , \tag{3}$$

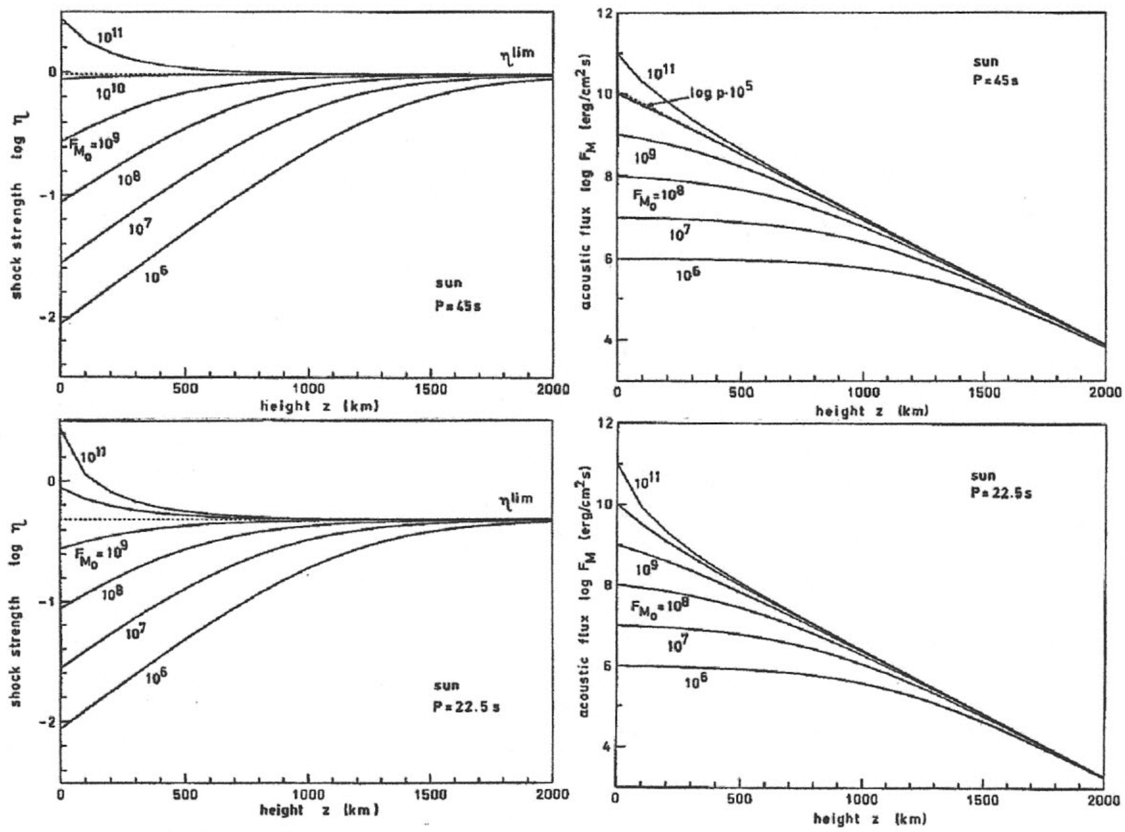

Figure 2. Left panels: Shock strength η versus height in a solar type non-ionizing, isothermal gravitational atmosphere with temperature $T = 6000\ K$. Sawtooth shock waves of different initial acoustic fluxes F_{Mo} (in $erg\ cm^{-2}s^{-1}$) and period $P = 45\ s$ (top) and $P = 22.5\ s$ (bottom) are shown. Right panels: The acoustic flux F_M versus height for the same waves. For the 45 s waves the curve $10^5 p_0$ is shown dashed.

where g is the gravitational acceleration. The refractive term dc_S^2/dz is small in the chromosphere but may become large in the transition layer.

For an isothermal, non-ionizing, gravitational atmosphere Eq. (3) shows that irrespective of the initial shock strength the shocks eventually reach a *limiting shock strength*

$$\eta^{lim} = \frac{\gamma\ g\ P}{(\gamma+1)\ c_S} \quad , \tag{4}$$

and with Eq. (1) a limiting wave flux

$$F_M^{lim} = \frac{1}{12}\ \frac{\gamma^3\ g^2\ P^2}{(\gamma+1)^2\ c_S}\ p_0 \quad . \tag{5}$$

Fig. 2 shows solutions of Eq. (3) for acoustic sawtooth shock waves with periods $P = 22.5,\ 45\ s$ and various initial fluxes F_{Mo} in a non-ionizing isothermal gravitational atmosphere of solar gravity. For most wave fluxes one initially has an exponential growth due to acoustic flux conservation which results from the first term on the RHS of Eq. (3). This growth is similar to that for acoustic waves in a gravitational atmosphere assuming flux conservation, $\rho_0 v^2 \sim \rho_0 c_S^2 \eta^2 = \ const.$ The increase in shock strength is eventually balanced by the increasing shock dissipation described by the last term of Eq. (3). Limiting strength is reached when η becomes constant and the flux proportional to the gas pressure p_0. For the unrealistic wave flux $F_M = 1 \cdot 10^{11} erg\ cm^{-2}s^{-1}$ which exceeds the total solar flux, it is seen that the shock dissipation outweighs the amplitude growth and that limiting strength is reached from above. It should be noted that, once limiting

strength has been reached the wave does no longer depend on the initial acoustic flux, that is, the wave has forgotten its origin. In the limiting strength state the acoustic wave thus becomes *independent of its wave generation process.*
A different form of Eq. (3) is

$$\frac{dF_M}{dz} = -\frac{1}{c_S^2}\frac{dc_S^2}{dz}F_M - \frac{F_M}{L_A} \quad , \tag{6}$$

where L_A is the acoustic damping length given by

$$L_A = \left(\frac{\gamma\, p_0\, c_S^3}{12\,(\gamma+1)^2}\right)^{1/2} F_M^{-1/2} P = \frac{c_S\, P}{(\gamma+1)\,\eta} \quad . \tag{7}$$

From Eq. (4) it is seen that upon reaching the limiting shock strength the damping length becomes equal to the scale height, $L_A^{lim} = H = c_S^2/\gamma g$. Heating laws of the type (3) or (6) assume that the shocks have small amplitude such that the weak shock relations are satisfied and that radiation damping from the waves is small which close to the stellar surface can be a bad approximation (Ulmschneider 1988).

The validity of Eq. (4) for the prediction of the limiting shock strength has recently been investigated by Cuntz and Ulmschneider (1988) using nonlinear time-dependent wave calculations. They find that for short period waves in non- ionizing, isothermal atmospheres with constant gravity the value predicted by Eq. (4) is closely reached. The basic property of acoustic shock waves to reach a limiting strength is also maintained in non-isothermal, ionizing atmospheres with height dependent gravity. Eq. (4) predicts the limiting strength in these more realistic atmospheres only if the actual values of the sound speed and γ are used and if short period waves ($P \leq P_A/5$, c.f. Eq. 9) are considered. For longer period waves Eq. (4) increasingly underestimates the limiting strength.

The weak acoustic shock theory was used to compute coronal models of late-type and early-type stars and to study the propagation of weak acoustic shock waves guided by diverging magnetic fields through a static model of the solar chromospheric network and transition layer. For references of this work see NU.

3. Acoustic energy spectrum in late-type stars

Before discussing radiation damping and more realistic acoustic wave calculations a discussion of the acoustic wave spectrum is in order. For the sun, in principle, the spectrum of acoustic waves can be directly observed. For a recent review of such observations see Deubner (1988, 1991, this volume). The problem with the observation of the propagating acoustic wave spectrum is twofold: First, fluctuations in the earth's atmosphere (seeing) degrades the solar signal. Second, there are observational limitations in the solar atmosphere. Acoustic waves are observed as velocity or temperature fluctuations in spectral lines. The spectral lines in the solar atmosphere are formed over height intervals where the line contribution function is appreciable. With a sound speed of 7 km/s, the wavelength of acoustic waves with $P = 45$ s and shorter, become smaller than the typical width of 300-400 km of the line contribution function and thus short period Doppler shifts or intensity fluctuations can no longer be measured. Using refined Fourier methods to analyze their data Endler and Deubner (1983) succeeded in removing most of the influence of the earth's atmosphere (see also Ulmschneider 1990). However the observational detection limit of acoustic waves posed by the contribution function can not be removed. Endler and Deubner (1983) find that the short period detection limit of acoustic waves is near $P = 40$ s. Fig. 5 from Ulmschneider (1990) shows directly observed short period acoustic fluxes by Deubner (1988). Unfortunately, theoretical calculations indicate that the limit $P = 40$ s is close to the period where the maximum of the acoustic wave spectrum is expected. The direct solar observation thus should detect only the long period fraction of the total acoustic power.

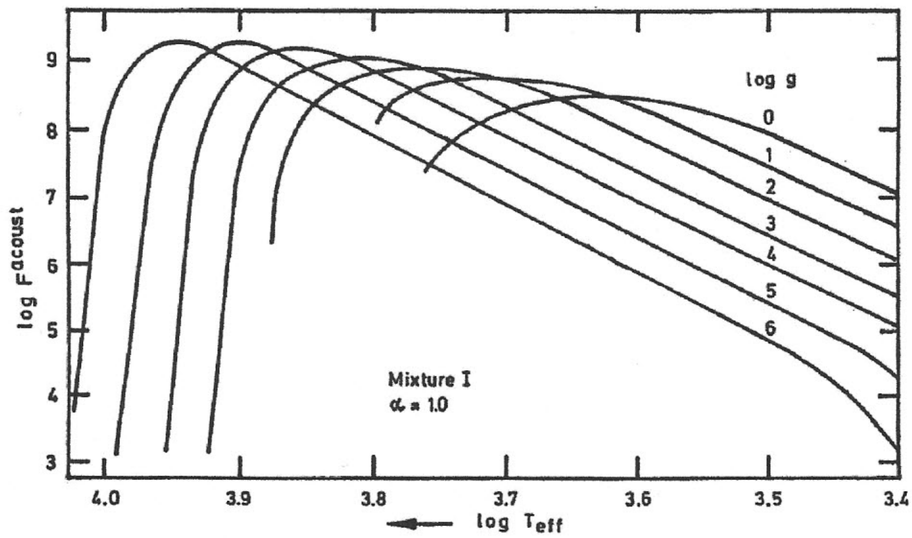

Figure 3. Theoretical acoustic energy fluxes generated in stellar surface convection zones versus log T_{eff} with log g as parameter after Bohn (1984).

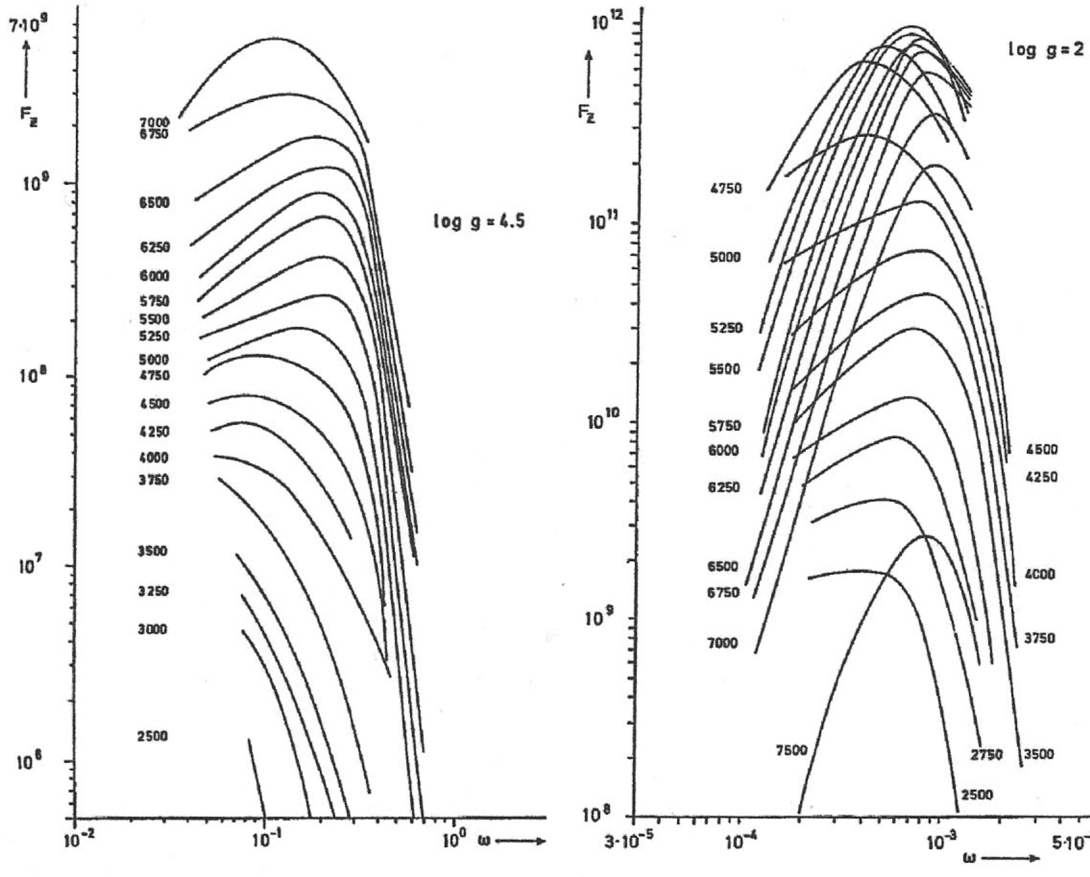

Figure 4. Acoustic frequency spectra for giant and dwarf stars of given gravity and T_{eff} after Bohn (1981). F_z is in $erg\ cm^{-2}\ s^{-1}\ Hz^{-1}$, $\omega = 2\pi/P$ in Hz.

Because of these difficulties our knowledge of the acoustic spectrum comes mainly from theoretical computations. Theoretical acoustic wave generation calculations have recently been reviewed by NU and by Musielak (1991, this volume). We therefore discuss only the most recent computations by Bohn (1981, 1984). Despite of some inaccuracies and difficulties with these computations I feel that they show general trends which permit insight in the basic behaviour of the acoustic wave generation and the acoustic spectrum.

Bohn's (1981, 1984) computations on basis of models of stellar convection zones depend on three parameters: effective temperature T_{eff}, gravity g, and the ratio α of the mixing length to the pressure scale height. Fig. 3 shows Bohn's acoustic flux results for $\alpha = 1.0$. It is seen that for given gravity the fluxes increase strongly with increasing T_{eff}. At high T_{eff} there is a cut-off, where the convection zones cease to exist. In addition, Fig. 3 shows that for given T_{eff} the acoustic fluxes increase strongly with decreasing gravity, that is, when going to giant stars. This is explained as follows. In fairly efficient convection zones the total stellar flux is carried mainly by the convective flux, $\sigma T_{eff}^4 \approx \rho u^3$, where u is the convective velocity. Moreover, the acoustic energy generation depends on a high power of the convective velocity. For given g the increase of T_{eff} towards earlier-type stars thus means increased u and thus an increased acoustic flux. Similarly, acoustic fluxes are increased if for given T_{eff} the gravity decreases and the thinner atmosphere is forced to carry the same amount of total flux by increasing the convective velocity u. Note that these arguments are very general and do not depend on the detailed computational procedure. In addition these considerations tell that the main contribution to the acoustic flux arises in the stellar surface layers near the top of the convection zone. Moreover, Fig. 3 shows that for dwarfs the maximum of the acoustic energy production occurs near the F-stars. Bohn (1981, 1984) has given a numerical fit to his flux computations:

$$F_M = 1.4 \cdot 10^{26}\ T_{eff}^{9.75} g^{-0.5} \alpha^{2.8} , \qquad (8)$$

which should be treated with caution due to the inaccuracies of his computations, but allows to make order of magnitude estimates of the acoustic flux. Fig. 4, taken from Bohn (1981) shows the acoustic frequency spectra of main-sequence and giant stars and it is seen that the spectra extend roughly over the range $\omega_A < \omega < 10\ \omega_A$, where ω_A (c.f. Eq. 9) is the acoustic cut-off frequency. This frequency range is explained as follows. The cut-off frequency is the local resonance frequency of the atmosphere below which acoustic waves can not propagate. The high frequency end of the spectrum is caused by the inability of the relatively slow convective velocity fluctuations to produce high frequency power. In addition, acoustic waves with high frequencies, due to the large density gradient at the stellar surface, would quickly form shocks and dissipate in the low photosphere, with the consequence that an extended high frequency range of waves could not contribute to the chromospheric heating. Similarly as above, these arguments are quite general and do not depend on the detailed computational procedure. Bohn's (1981) acoustic frequency spectra thus show a period range of $P_A/10 < P < P_A$, with the acoustic cut-off period P_A given by

$$P_A = \frac{2\pi}{\omega_A} = \frac{4\pi c_S}{\gamma g} , \qquad (9)$$

and a maximum of the spectrum roughly at the period $P_A/5$ as seen in Fig. 4. With the necessary caution we are thus able to deduce from theory the magnitude and frequency of the acoustic wave spectrum in late- type stars. Note that these acoustic fluxes depend only on T_{eff}, g and α. Similar calculations in early-type stars where acoustic fluxes are expected to be proportional to the stellar radiation flux are not available and the acoustic spectra are not known.

4. Limiting flux and solar chromospheric losses

Combining the knowledge of the acoustic spectrum with the limiting strength concept allows to predict the amount of acoustic heating in the middle and higher stellar chromospheres where the shocks have grown to limiting strength. The advantage of the limiting strength behaviour is, as discussed above, that it is independent of the acoustic energy generation in the convection zone and thus allows very definite theoretical predictions. Assuming that the acoustic spectrum

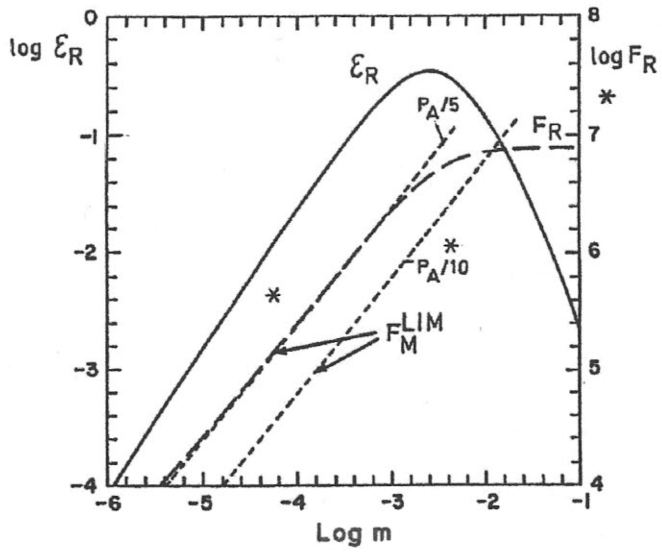

Figure 5. Comparison of the theoretical limiting acoustic flux F_M^{lim} $(erg\ cm^{-2}s^{-1})$ with the solar chromospheric radiative loss flux F_R determined empirically by Anderson and Athay (1990). ϵ_R $(erg\ cm^{-3}s^{-1})$ is the empirical net radiative cooling rate. $P_A/5$ and $P_A/10$ label different assumptions as to the acoustic frequency spectrum. Star symbols show directly observed acoustic fluxes by Deubner (1988).

can be represented by a wave with a period near the maximum of the spectrum, that is, by a wave with a period $P = P_A/5$ one finds combining Eqs. (4), (5), (9)

$$\eta^{lim} = \frac{1}{5}\frac{4\pi}{\gamma+1} = 0.94 \quad , \tag{10}$$

$$F_M^{lim} = \frac{4\pi^2}{75}\frac{\gamma}{(\gamma+1)^2}\, c_S\, p_0 \;\approx\; .123\, c_S\, p_0 \;\approx\; 10^5\, p_0 \quad (erg\ cm^{-2}s^{-1}) \quad . \tag{11}$$

A first conclusion from these two equations by Ulmschneider (1989) is that the atmospheres of late-type stars, independent of T_{eff} and g, are permeated by limiting strength acoustic shock waves of roughly identical strength. A second conclusion is that the limiting strength acoustic fluxes in these stars are proportional to the gas pressure. Note that with a strength of $\eta = 0.94$, that is, $v_m = 0.47\ c_S$ the shocks are no longer weak and the above equations lose their validity. But time-dependent calculations discussed below show that the weak shock predictions for these wave periods are still surprisingly good although the weak shock approximation is no longer strictly valid.

For the sun with $c_S(T_{eff})$ and $P = P_A/5 = 43\ s$ one finds $F_M^{lim} = 9.7 \cdot 10^4\ p_0$. This can be compared with recent empirical determinations of the chromospheric radiation loss flux F_R. Fig. 5, adapted from Anderson and Athay (1990) and Ulmschneider (1990) shows such a comparison of F_R and F_M^{lim}. In addition, to show the influence of the uncertainty of the acoustic spectrum, a flux F_M^{lim} with $P = P_A/10 = 22\ s$ has been plotted. The interesting result is, that both the magnitude and the slope of F_R are well reproduced in the middle and upper chromospheric regions where the waves are presumed to have reached limiting strength.

5. Radiation damping

The second basic process which affects acoustic waves in stellar atmospheres is the exchange of radiation, either by radiation damping or radiative amplification. Whether radiative damping or amplification prevails in acoustic waves in late-type stars depends on the strength of the radiation

Table 1. Radiative damping time t_{rad} at the stellar surface and acoustic wave period $P = P_A/5$ for stars of given spectral type (S.T.), luminosity class (L.C.), T_{eff} and g, adopted from Ulmschneider (1988).

$Sp.T.$	$T_{eff}(K)$	$\log g$	$t_{rad}(s)$	$P(s)$	P/t_{rad}	$L.C.$
F0	7700	1.7	8	$2.4 \cdot 10^4$	3200	
G0	5550	1.3	80	$5.2 \cdot 10^4$	660	I
K0	4420	.94	450	$7.8 \cdot 10^4$	260	
M0	3650	.14	3000	$6.8 \cdot 10^5$	220	
G0	5850	2.9	12	$1.4 \cdot 10^3$	114	
K0	4750	2.1	91	$7.6 \cdot 10^3$	84	III
M0	3800	1.3	740	$4.4 \cdot 10^4$	58	
F0	7200	4.3	.9	60	66	
G0	6030	4.4	2.5	50	20	V
K0	5250	4.5	5.1	36	8	
M0	3850	4.6	29	24	.8	

field and on the behaviour of the emission or absorption coefficients. For a solar acoustic wave calculation Ulmschneider et al. (1978) found that in the low photosphere, where the optical depth was large, radiative amplification occured, while in the higher optically thin layers of the photosphere and chromosphere radiative damping occured. Radiative amplification is a regular feature in early-type stars as shown for instance by Wolf (1987) and Owocki et al. (1988). In the optically thin outer layers of late-type stars, radiative damping prevails. The importance of the effect of radiation damping on acoustic waves in optically thin cases can be discussed by considering the radiative damping time t_{rad} (Ulmschneider 1988). This characteristic time describes the energy loss of the acoustic wave by radiation and has recently been critically discussed by Schmitz (1990). After Schmitz (1990) the radiative damping time is given by

$$t_{rad} = \frac{2.5 \, c_v}{16 \, \kappa \sigma \, T^3} \quad , \tag{12}$$

where c_v is the specific heat at constant volume, κ the Rosseland mean opacity and σ the Stefan Boltzmann constant. Radiation damping is important for the acoustic wave if $t_{rad} < P$. Tab. 1 compares the two important timescales, where for the acoustic wave period the value $P = P_A/5$ valid for the maximum of the acoustic spectrum has been taken. The ratio P/t_{rad} shows that depending on the type of star, the acoustic waves suffer very differently from radiation damping. The largest radiation damping occurs in supergiants, while the giants also greatly suffer but much less so than the supergiants. The dwarf stars in turn suffer much less than the giants.

The radiative damping times of Tab. 1 are computed for the stellar surface at optical depth unity. t_{rad} increases rapidly with height in the stellar atmosphere due to the strong pressure and temperature dependence of the Rosseland opacity, which mainly consists of H^-. In the sun, for instance, the height at which $t_{rad} = P$ lies near 200 km. The zone in which $t_{rad} < P$ is called *radiative damping zone* and is the atmospheric region where the behaviour of the acoustic wave is dominated by radiation damping, that is, where the growth of the wave amplitude is greatly inhibited. Even without a detailed model it is clear from Tab. 1 that supergiants have extensive-, giants large- and dwarfs small radiation damping zones.

6. Strong shock treatments

In acoustic wave calculations where shocks are no longer weak, two types of difficulties occur. The first is that for larger amplitude waves radiation effects can no longer be neglected, because the most important seat of emission usually occurs in the region immediately behind the shock. In weak shock cases usually only the cooling, averaged over the wave is important such that heating and cooling can be decoupled. One uses Eqs. (3) or (6) to compute the development of the shock heating, while separate cooling laws which involve only the wave-averaged atmospheric quantities are taken to obtain energy balance. In the strong shock case this decoupling is not easily done as the shape of the post-shock region determines the radiation loss and the radiation in turn determines the future development of the shock.

The second difficulty arises from the fact that for larger wave amplitude the weak shock approximation in Eq. (2) becomes bad. With the shock Mach number $M_S \equiv (U - u_1)/c_1$, where U is the shock speed, u_1 and c_1 the velocity and sound speed in front of the shock one has (c.f. Landau Lifshitz 1959, p.331):

$$\frac{p_2}{p_1} = \frac{2\gamma M_S^2 - \gamma + 1}{\gamma + 1} \quad , \qquad \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_S^2}{(\gamma-1)M_S^2 + 2} \quad . \tag{13}$$

Using these shock relations in Eq. (2) to compute the heating one can show that the weak shock approximation, which is valid for small η and which leads to the η^3 term on the RHS of Eq. (2) grows too rapidly for larger η. For instance for $\eta = .075,\ .15,\ .44,\ .96,\ 1.3$ the ratio of the weak shock heating to the true heating is $\epsilon_{MW}/\epsilon_M = 1.1,\ 1.2,\ 1.8,\ 3.2,\ 4.6$, respectively.

In order to construct atmospheric models in a similar way as for weak shocks, namely employing separate wave-averaged heating and cooling laws, not only the unapproximated Eq. (2) must be used, but additional information about the behaviour behind the shock front has to be employed. A powerful method is to adopt the principle of ***shape similarity invariance*** found experimentally. This principle states that the shape of the shock stays essentially the same during propagation. In this approach based on the work of Brinkley and Kirkwood (1947) and Bird (1964), the amount of shock heating obtained depends on the way by which the post shock state relaxes. For references of work using the strong shock approach see NU.

7. Time-dependent wave calculations

In realistic situations of stellar atmospheric wave calculations one is not only interested in shock propagation, but also in the entire process of how acoustic waves behave in the presence of radiation damping, how shocks form and even how shocks overtake one another. To describe these processes and to take into account the detailed mutual interaction of wave shape and radiation, time-dependent radiation hydrodynamic methods have to be employed for stellar acoustic wave computations.

However, this does not mean that time-independent computations are obsolete. In many situations, time-independent methods give a simple and sufficiently accurate description of the shock wave propagation. For instance in late-type giant stars where computations in extended envelopes are necessary for the study of mass loss, the acoustic wave propagation can usually not be included because of the small wavelength compared to the extent of the computational domain. Here it is advantageous to do the acoustic computation with time-independent-, and the long period hydrodynamic computation with time-dependent methods (Gail et al. 1990).

So far time-dependent acoustic wave calculations were done mainly to obtain chromospheric models for the sun and other late-type stars. There are also other applications. Klein et al. (1976) investigated Lyman continuum emitting transients in A-star atmospheres. Wave propagation along coronal loops was discussed e.g. by Mariska and Boris (1983) or McClymont and Canfield (1983). For reviews of time-dependent radiative (magneto-) hydrodynamic wave calculations in stellar atmospheres see Ulmschneider and Muchmore (1986), and NU.

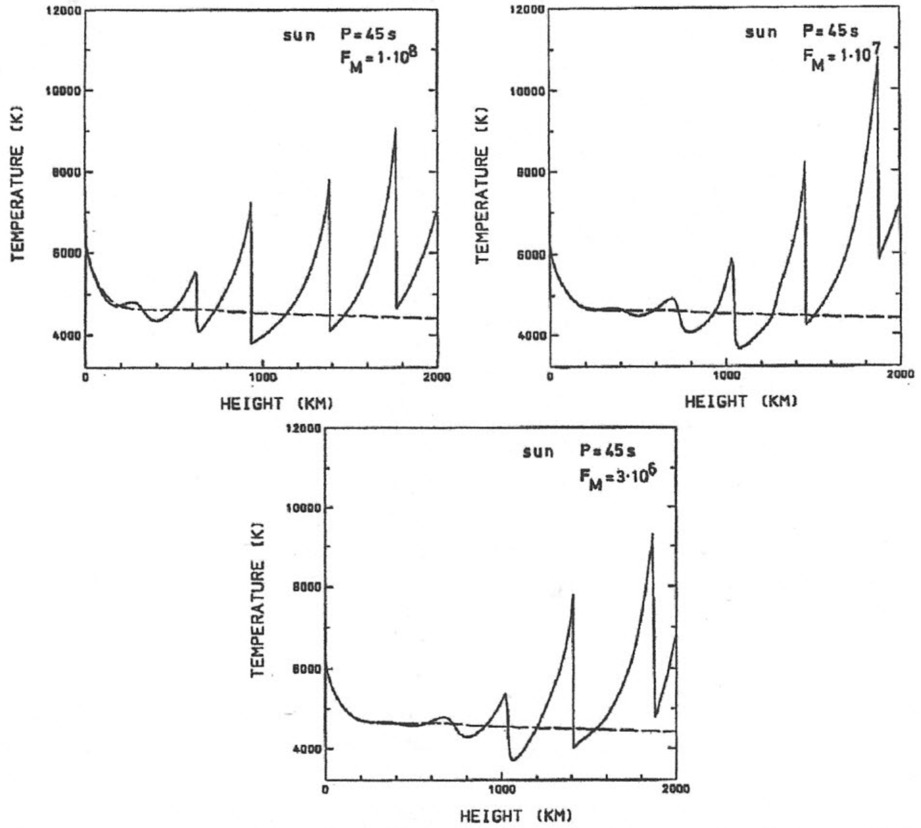

Figure 6. Temperature distributions of acoustic wave calculations with $P = 45\ s$ and three different initial wave fluxes F_{Mo} $(erg\ cm^{-2}s^{-1})$ in the solar atmosphere. The initial radiative equilibrium atmosphere is shown dashed.

Typical time-dependent calculations of the acoustic wave propagation in the solar chromosphere were done by Ulmschneider et al. (1978, 1987), Schmitz et al. (1985), using the method of characteristics. For the radiation treatment a NLTE H^- continuum and a scaled two level atom NLTE Mg II k-line computation were used, assuming complete redistribution for the line. The restriction to only one single chromospheric line was necessary to save computation time and at the same time take into account the important chromospheric losses in the many Ca II, Mg II and Fe II lines. For this, the radiative cooling of the Mg II k-line was scaled up to account for the total chromospheric line losses. The inclusion of the hydrogen lines and continua together with the treatment of ionization is a formidable problem which is currently under investigation by several groups of workers. At the present time, where these calculations are not yet available, acoustic wave calculations can not be extended to the high chromosphere and lower transition layer, although it is highly desirable to carry these calculations all the way into coronal loops.

A series of solar acoustic wave calculations with period $P = 45\ s$ and different wave energy fluxes F_{Mo} by Rammacher and Ulmschneider (1991) are shown in Fig. 6. For each wave calculation it is seen that after the wave crosses the radiation damping zone, which extends from the surface to about 200 km height, the wave amplitude grows rapidly and shocks form. The shocks in a distance of about a wavelength grow into a fully developed sawtooth shape and reach limiting strength. Independent of the initial wave flux, this limiting strength is the same for all three wave calculations. Despite the fact that one now has a non-isothermal radiating atmosphere, there is still the tendency to reach limiting strength. This shows that the limiting strength behaviour is a basic property of acoustic waves in gravitational atmospheres. With $T_2/T_1 \approx 1.91$ and Eq. (13) the calculations of Fig. 6 yielded a shock strength of $\eta \approx 1.15$. It is interesting that despite of the above mentioned difficulty with the validity of the weak shock theory, η is close to the predicted value $\eta^{lim} = 0.92$ from Eq. (4). In the discussion of Eq. (7),

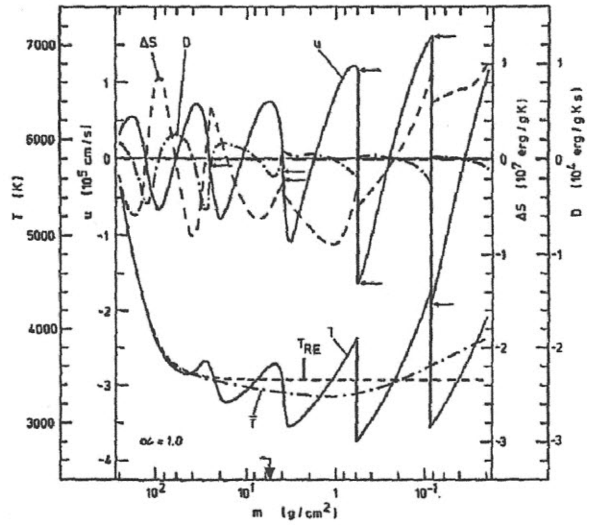

Figure 7. Acoustic wave calculation with $P = 1.4 \cdot 10^4\ s$ and an initial wave flux $F_{Mo} = 2.5 \cdot 10^7\ erg\ cm^{-2} s^{-1}$ for Arcturus after Ulmschneider et al. (1979). Horizontal arrows show the development of the shock, the vertical arrow the extent of the radiation damping zone.

valid for weak shocks, it was noted that in the limiting strength state the damping length L_A^{lim} becomes equal to the scale height H. Time-dependent calculations of Schmitz et al. (1985) find that for the limiting case $L_A^{lim} \approx 1.4H$.

The variation of F_{Mo} as shown by Fig. 6 mainly affects the height of shock formation. Large F_{Mo} leads to shock formation at low height, smaller F_{Mo} leads to shock formation at progressively greater height. The more energetic wave dissipates its energy deeper in the atmosphere such that at great height the same limiting strength shock heating occurs regardless of the initial acoustic flux. Another nonlinear property of acoustic waves, as shown in Fig. 6, is that they produce a depression of the average temperature at the temperature minimum region below the initial radiative equilibrium value (Ulmschneider et al. 1978). This depression is explained as follows. Small amplitude waves oscillate around the radiative equilibrium temperature. Here the excess emission at the wave crests is compensated by an excess absorption at the wave troughs. However, for large amplitude acoustic waves the wave crests, due to the nonlinearity of the Planck function have a disproportionally large excess emission compared to the much smaller absorption at the wave troughs. To reach a dynamical steady state, where emission and absorption balance in the time average, the wave therefore must oscillate around a mean temperature which is lower than the radiative equilibrium temperature. Kalkofen et al. (1984) showed that despite of this depression the chromospheric line emission is enhanced, which by an outside observer may be interpreted as an apparent temperature enhancement.

By the same methods as above, chromospheric models for late-type stars other than the sun were constructed. Fig. 7 shows an acoustic wave calculation for Arcturus. This giant star, as discussed above, has a much more extended radiation damping zone compared to the sun. Unlike the solar case, in this star the point of shock formation as seen in Fig. 7 occurs in the radiation damping zone. Consequently, as indicated by horizontal arrows in Fig. 7, the shock does not grow until the radiation damping zone has been passed. Here unlike the solar case the temperature minimum, which is formed when significant shock heating raises the temperature, does not lie near the height of shock formation. In Arcturus, the height of the temperature minimum is marked by the end of the radiation damping zone. The different types of chromospheric temperature rises in cases of the sun and Arcturus have been used to classify chromospheres which start near the point of shock formation as *S-type chromospheres*, and chromospheres which begin at the end of the radiation damping zone as *R-type chromospheres*.

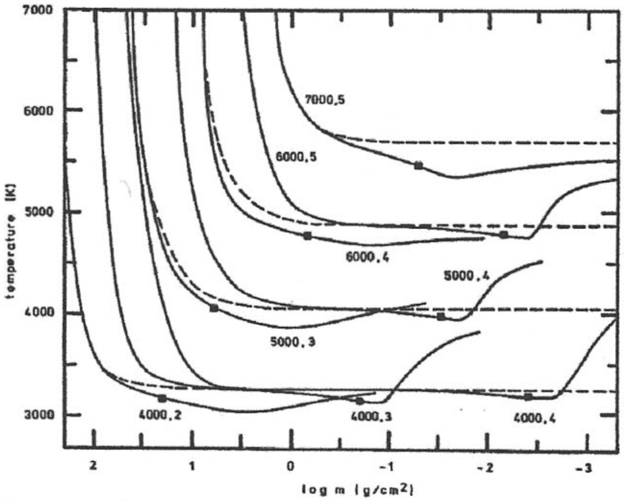

Figure 8. Theoretical mean chromospheric temperature distributions after Schmitz and Ulmschneider (1981). The curves are labelled by T_{eff} and log g. Solid squares indicate points of shock formation, circles heights of the radiative damping zone.

Fig. 8 by Schmitz and Ulmschneider (1981) shows a series of acoustic wave chromospheric model computations which use a grey LTE radiation treatment. In Fig. 8 the time-averaged temperatures are shown, together with the heights of shock formation and the extent of the radiative damping zones. It is seen that the S-type chromospheres have a much steeper chromospheric temperature rise compared to the R-type chromospheres. As the radiation treatment has been considerably improved in the last 10 years, and better estimates for the initial acoustic flux possibly are soon available (c.f. Musielak 1991, this volume), it is highly desirable to repeat calculations of this type to get insight in the systematic variation of acoustically heated chromospheres. It should be noted that results of the type shown in Fig. 8 are the ultimate aim of our effort to identify the heating mechanisms. A correctly identified heating mechanism would allow to connect the chromospheric structure to the interior structure of late-type stars. Such a physical connection must in principle be possible, because the chromospheres are completely determined from the physical state of the stellar interior.

To study the effect of gravity Fig. 9 shows chromospheric acoustic wave calculations for two stars with log $g = 3,\ 5$ but with the same $T_{eff} = 5012\ K$. Initial acoustic fluxes $F_{Mo} = 2.0 \cdot 10^8,\ 2.5 \cdot 10^7 erg\ cm^{-2} s^{-1}$ and wave periods $P = P_A/10 = 250,\ 2.5\ s$ from Eq. (9) were used for the giant and dwarf star, respectively. For both stars the limiting shock strength behaviour is found despite the fact that the atmospheres are not isothermal and the waves are radiating. This is similar as for our solar case. In addition it was found that the limiting strength in the two stars is identical, as predicted by the weak shock theory which shows that the conclusions from this theory are more general. In the giant the radiation damping zone (marked by an arrow) is extended and F_M decreases, initially due to radiation damping, and later because of shock dissipation. The wave amplitude grows very slowly in the radiation damping zone. In the dwarf, F_M initially is conserved due to the small radiation damping zone and decreases later because of shock dissipation (compare with Fig. 2). Yet despite the fact that the giant initially has an order of magnitude more acoustic flux, it ends up with about ten times less flux at similar optical depth as soon as the limiting strength flux has been reached. This is due to the fact that when limiting strength is reached the acoustic flux is proportional to the gas pressure which is an order of magnitude smaller in the giant. Note that at limiting strength $F_M \sim p$ as predicted by the weak shock theory.

So far only calculations of a single wave period have been discussed. Increasing the wave period one finds that the radiation damping in the photosphere is smaller, that the shock formation height is increased, but that the shocks become stronger and consequently show much greater post-shock emission. Ideally one should compute a spectrum of acoustic waves, but this

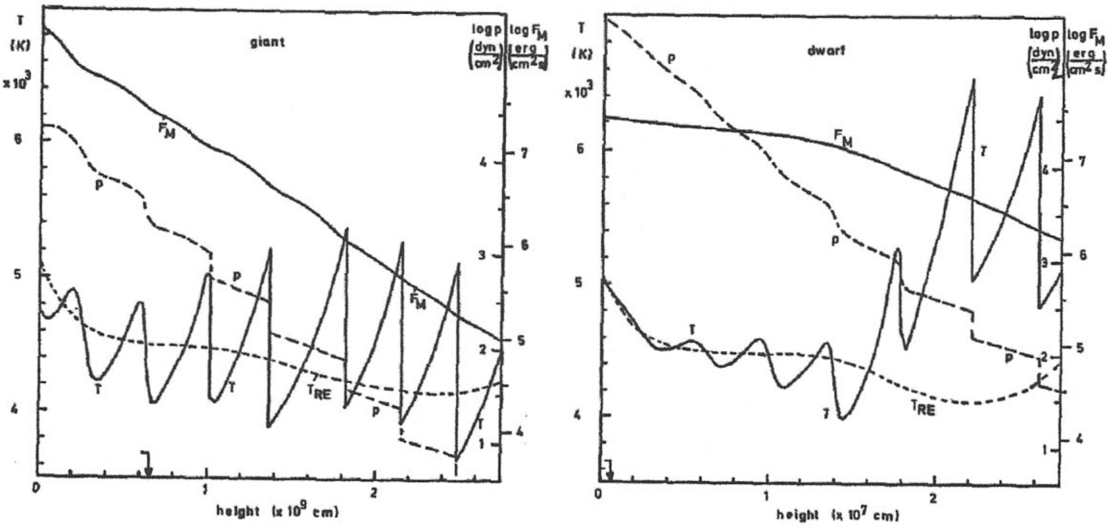

Figure 9. Acoustically heated theoretical chromosphere models of a giant and a dwarf star after Ulmschneider (1989).

needs much more computation time if time-averaged results are desired. Cuntz (1989, 1991, this volume) discusses acoustic wave calculations in giant stars which employ a stochastically changing acoustic spectrum. He finds that the process of shock overtaking produces long period shocks which grow rapidly in strength by cannibalizing on other shocks. These processes result in transient events.

The long period acoustic waves, the 5 min- and 3 min- oscillations, which are an outstanding observational effect on the sun are not candidates for solar chromospheric heating because they are standing waves and the observed phase shift of 90^o between velocity and temperature fluctuations precludes that these waves form shocks in the chromosphere. Long period acoustic waves other than the short period waves are the result of radial and non-radial vibrations of the star. In late- type giant stars this acoustic wave type eventually leads to large scale pulsations, which in Mira stars and Semiregular Variables are observed to produce extensive shock heating in the outer layers. The observed oscillation periods (c.f. Eaton et al. 1990) are in the range $5 - 50P_A$. This form of heating, also called *pulsational heating,* is a type of acoustic heating as the dissipation by hydrodynamic shocks is identical to that for short period acoustic waves. Long period acoustic waves are thought to be instrumental for generating mass loss in late-type giant stars (Cuntz 1989).

8. Relation to magnetic heating, basal flux stars

About ten years ago observations using the IUE and Einstein satellites showed that stars with the same T_{eff} and gravity often have greatly different UV and X-ray emission. It was found that this emission variability is tightly correlated with the stellar rotation and the magnetic field coverage (see Schrijver 1991 and Zwaan 1991, both this volume). As for given T_{eff} and g the acoustic wave generation in the convection zone results in a fixed value of the acoustic flux which can not explain the observed emission variability, it was concluded that the chromospheric and coronal emission in typical stars is not due to acoustic heating but to some magnetic heating mechanism. With the accumulation of more observational data this pure magnetic heating picture must now be modified.

Recently Schrijver (1987) and others found that by subtracting from the measured chromospheric Ca II emission flux from stars of a given T_{eff} the lowest observed emission flux from stars at this T_{eff}, the correlation between the observed X-ray flux and the Ca II emission flux is considerably improved. This was taken as indication that there are two basic components of the chromospheric heating of late-type stars: a ***nonmagnetic heating component*** which is independent of rotation and which constitutes a basic contribution depending only on T_{eff} and possibly slightly on g, and a rotation-related ***magnetic heating component*** which is usually the dominant contribution. The heating by the nonmagnetic component leads to a ***basal chromospheric emission flux*** which very likely is heating by acoustic shock waves. This basal flux constitutes an intrinsic lower limit of chromospheric emission, observable locally in nonmagnetic areas on the sun and globally in stars of very low rotation rate (the so called basal flux stars) but for faster rotating stars is usually greatly exceeded by the more energetic magnetic heating component. It should be noted that this magnetic heating component in the chromosphere very likely consists of acoustic-like slow-mode or longitudinal mhd tube waves (Ulmschneider 1986, Stein 1991, this volume) which on the sun may be difficult to differentiate from acoustic waves in non-magnetic areas.

The idea that in late-type stellar chromospheres two components, a nonmagnetic, very likely acoustic component and a magnetic component are at work, can explain several other observations. From the gravity dependence of the acoustic energy generation (see Fig. 3) one might expect a higher basal flux limit for giants than for dwarfs. Observations actually show the opposite, that the dwarfs appear to have a slightly higher basal flux limit than the giants (Schrijver et al. 1989). It has to be kept in mind that in a comparison of the generated acoustic flux and the Ca II emission, different stellar layers are involved. The acoustic energy generation occurs at the top of the convection zone, while the Ca II emission arises from chromospheric heights. The lower basal flux limit in giants may be explained by the greater radiation damping of acoustic waves and the limiting shock strength behaviour of the acoustic waves in these stars which, compared to the dwarfs, leads to a lower chromospheric acoustic flux due to the lower gas pressure in giant atmospheres. In addition, the low chromospheric variability of the F-stars may be explained as the superposition of a maximum acoustic heating contribution and a given variable magnetic heating component. Finally observations show that in late-type giant stars the emission variability decreases rapidly toward later spectral type and there becomes a low basal emission (Middelkoop 1982). This is explained as due the low rotation rate from these stars resulting from angular momentum conservation during the large evolutionary increase in radius and from angular momentum loss by massive stellar winds. Stellar coronae of late-type stars are very likely not heated by short period ($P < P_A$) acoustic waves as discussed by Hammer and Ulmschneider (1991, this volume) which supports the assumption, used in the separation of the magnetic and nonmagnetic heating components, that the observed X-ray emission is from magnetically heated coronae.

9. Conclusions

From the above we may draw the following conclusions:
1. Acoustic waves heat by hydrodynamic shocks and are a viable heating mechanism for late-type stars. The acoustic wave energy spectrum generated in the stellar convection zone is in the period range $P_A/10 < P < P_A$ and increases with increasing T_{eff} and decreasing g.
2. Acoustic waves are a dominant chromospheric heating mechanism, locally in nonmagnetic areas on stars and globally in stars of very low rotation rates which do not have appreciable magnetic fields. For more rapidly rotating stars with significant magnetic fields acoustic heating constitutes a weak background.
3. Acoustic shock waves show the tendency to grow to a limiting strength which is roughly similar in all late-type stars.
4. The dissipation by limiting strength acoustic shock waves satisfies roughly the solar chromospheric energy requirements. Acoustic waves in the right frequency range and consistent with the chromospheric energy requirements are directly observed on the sun.
5. For very late-type stars like Miras and Semiregular Variables the pulsational heating by hydrodynamic shock waves with periods ($P >> P_A$) is an acoustic heating mechanism which acts in addition to the short period ($P < P_A$) acoustic heating.

6. Acoustic wave heating also occurs in early-type stars where small disturbances are amplified by the strong radiation field.

7. The acoustic-like slow-mode mhd waves or longitudinal tube waves in strong magnetic fields also dissipate by hydrodynamic shocks. It is customary, however, to discuss these waves not with the acoustic- but with the magnetic heating mechanisms.

Acknowledgement This work was supported by DFG project UL 57/11-2.

References

Anderson, L.S., Athay, R.G..: 1990, Astrophys. J., in press

Biermann, L.: 1946, Naturwiss. **33**, 118

Biermann, L.: 1948, Z. Astrophys. **25**, 161

Bird, G.A.: 1964, Astrophys. J. **139**, 675

Bohn, H.U.: 1981, Ph.D. Thesis, Univ. Würzburg, Germany

Bohn, H.U.: 1984, Astron. Astrophys. **136**, 338

Bray, R.J., Loughhead, R.E.: 1974, The Solar Chromosphere, Chapman and Hall, London, England

Brinkley, S.R. Jr., Kirkwood, J.G.: 1947, Phys. Rev. **71**, 606

Cuntz, M., Ulmschneider, P.: 1988, Astron. Astrophys. **193**, 119

Deubner, F.-L.: 1988, in Pulsation and Mass Loss in Stars, R. Stalio, L.A. Willson Eds., Kluwer Academic Publ., p. 163

Edlén, B.: 1941, Ark. Mat. Astr. Och. Fys. **28B**, No. 1

Eaton, J.A., Johnson, H.R., Cadmus, R.R.: 1991, Astrophys. J., in press

Endler, F., Deubner, F.-L.: 1983, Astron. Astrophys. **121**, 291

Gail, H.-P., Cuntz, M., Ulmschneider, P.: 1990, Astron. Astrophys., in press

Herbold, G., Ulmschneider, P., Spruit, H.C., Rosner, R.: 1985, Astron. Astrophys. **145**, 217

Kalkofen, W., Ulmschneider, P., Schmitz, F.: 1984, Astrophys. J. **287**, 952

Klein, R.I., Stein, R.F., Kalkofen, W.: 1976, Astrophys. J. **205**, 499

Kuperus, M., Ionson, J.A., Spicer, D.S.: 1981, Ann. Rev. Astron. Astrophys. **19**, 7

Landau, L.D., Lifshitz, E.M.: 1959, Fluid Mechanics, Pergamon Press, London

Mariska, J.T., Boris, J.P.: 1983, Astrophys. J. **267**, 409

McClymont, A.N., Canfield, R.C.: 1983, Astrophys. J. **265**, 483

Middelkoop, F.: 1982, Astron. Astrophys. **113**, 1

Narain, U., Ulmschneider, P.: 1990, Space Science Reviews, in press

Owocki, S.P., Castor, J.I., Rybicki, G.B.: 1988, Astrophys. J. **335**, 914

Rammacher, W., Ulmschneider, P.: 1991, Astron. Astrophys., in preparation

Schmitz, F.: 1990 Astron. Astrophys. **229**, 177

Schmitz, F., Ulmschneider, P.: 1981, Astron. Astrophys. **93**, 178

Schmitz, F., Ulmschneider, P., Kalkofen, W.: 1985, Astron. Astrophys. **148**, 217

Schrijver, C.J.: 1987, in Cool Stars, Stellar Systems and the Sun, J.L. Linsky, R.E. Stencel Eds., Lecture Notes in Physics **291**, Springer, Berlin, Germany, p. 135

Schrijver, C.J., Coté, J., Zwaan, C., Saar, S.H.: 1989, Astron. Astrophys. **337**, 964

Schwarzschild, M.: 1948, Astrophys. J. **107**, 1

Ulmschneider, P.: 1970, Solar Phys. **12**, 403

Ulmschneider, P.: 1986, Advances in Space Res. **6**, 39

Ulmschneider, P.: 1988, Astron. Astrophys. **197**, 223

Ulmschneider, P.: 1989, Astron. Astrophys. **222**, 171

Ulmschneider, P.: 1990, in Cool Stars, Stellar Systems and the Sun, Astr. Soc. Pacific Conf. Ser. **9**, G. Wallerstein Ed., p. 3

Ulmschneider, P., Muchmore, D.: 1986, in Small Scale Magnetic Flux Concentrations in the Solar Photosphere, W. Deinzer, M. Knölker, H.H. Voigt Eds., Vandenhoeck and Ruprecht, Göttingen, Germany, p. 191

Ulmschneider, P., Muchmore, D., Kalkofen, W.: 1987, Astron. Astrophys. **177**, 292
Ulmschneider, P., Schmitz, F., Hammer, R.: 1979, Astron. Astrophys. **74**, 229
Ulmschneider, P., Schmitz, F., Kalkofen, W., Bohn, H.U.: 1978, Astron. Astrophys. **70**, 487
Wolf, B.E.: 1987, Astron. Astrophys. **179**, 371

Discussion

E. Priest: In a magnetic atmosphere I would prefer to say that there is a background of slow-mode mhd waves. Are your 'observed' acoustic waves really slow-mode waves?

Answer: I agree, but for stars you usually integrate over surface regions with and without magnetic fields. The contribution from the field-free regions constitutes the acoustic background. In my view the observations also include slow-mode waves which are essentially acoustic waves propagating along the field lines.

J. Pasachoff: Your models are for a plane-parallel atmosphere, whereas real chromospheres are better approximated by 10^5 cylinders?

Answer: Acoustic wave heating probably is fairly uniform over a stellar surface, unlike magnetic heating, which is extremely variable horizontally.

M. Kuperus: In a turbulent convective medium there is a large directional dependence in the sense that much more sound is generated in the direction of the convective velocity. In the solar case I calculated a ten times stronger sound production in the upwellings (Sol. Phys. **22**, 257, 1972). It demonstrates that the solar atmosphere does not have horizontally uniform sound production.

Answer: This is a very important point the consequences of which have not been looked at in detail.

S. Koutchmy: In the case of exploding granules there seems to be evidence for shocks propagating horizontally. Does this affect your picture?

Answer: Luckily reality is more complicated than the simple model of a theoretician. The horizontal acoustic wave propagation together with acoustic wave generation by horizontal flux tube motions have not been investigated in detail.

C. Lindsey: Regarding Koutchmy's point about waves propagating non- vertically: Most of these waves are created in the convection zone where the sound speed is several times that in the chromosphere. Don't these waves get refracted to nearly vertical by the time they arrive in the chromosphere?

Answer: The sound speed at the top of the convection zone is roughly the same as in the chromosphere. The intermediate low temperature region will indeed refract acoustic waves in the vertical direction.

J. Scudder: What is the role of conduction in the limiting strength picture?

Answer: Acoustic heating mainly applies to the chromosphere where thermal conduction and viscosity are 5 orders of magnitude less important for the heating than shock heating. This is different for the transition layer.

B. Haisch: In the coolest active dwarfs the irradiation of the chromosphere by coronal X-rays could be a major heating source. Would this change your acoustic heating picture?

Answer: It is now clear that acoustic and magnetic heating mechanisms both exist. Coronae are almost certainly heated magnetically. Therefore both mechanisms will contribute.