Solar Magnetic Fields

Proceedings of the international conference
held in Freiburg, Germany, June 29–July 2, 1993

Edited by
MANFRED SCHÜSSLER
and
WOLFGANG SCHMIDT
Kiepenheuer-Institut für Sonnenphysik, Freiburg, Germany

CAMBRIDGE
UNIVERSITY PRESS

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011–4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

First published 1994

Printed in Great Britain at the University Press, Cambridge

A catalogue record for this book is available from the British Library

ISBN 0 521 46119 7 hardback

Oblique shocks in longitudinal-transverse mhd tube waves

Y. ZHUGZHDA[1], P. ULMSCHNEIDER[2], V. BROMM[2]

[1] *Laboratory of Cosmic Electrodynamics, IZMIRAN, 142092 Troitsk, Moscow Region, Russia*
[2] *Institut für Theoretische Astrophysik, Universität Heidelberg, Im Neuenheimer Feld 561, 69120 Heidelberg, Germany*

Abstract

The one-dimensional longitudinal-transverse wave calculations of Ulmschneider et al. (1991) in magnetic flux tubes are extended to include shocks. We find that in the most general case oblique kink shocks occur where the shock surface is symmetric with respect to the kink angle. The shocks suffer strong non-linear mode-coupling and propagate with a common speed.

1. Introduction

In a stellar atmosphere, consider a thin, roughly vertically directed magnetic flux tube with a field strength B, embedded in a non-magnetic external medium. At some height we assume an oblique shock, which is supposed to propagate towards greater height. Let the variables in the region immediately in front and behind the shock be denoted by indices 1 and 2, respectively. The tube behind the shock points in the direction of unit vector $\hat{\mathbf{l}}_2$ and has the cross-section A_2, while in front of the shock, it points in the direction $\hat{\mathbf{l}}_1$ and has the cross-section A_1. The physical state inside the tube behind the shock is given by the gas velocity $\mathbf{u}_2$, the density ρ_2, the gas pressure p_2 and the magnetic field strength B_2. The gas pressure external to the tube is p_e. We assume that p_e depends only on height and not on time. In front of the shock the physical state in the tube is given by similar quantities with index 1. Note that $\mathbf{u}_1$, $\mathbf{u}_2$ are not necessarily directed along the tube. The normal of the shock $\hat{\mathbf{e}}_n$ is assumed to be inclined by an angle φ_2 against the tube axis $\hat{\mathbf{l}}_2$ and by an angle φ_1 against $\hat{\mathbf{l}}_1$, such that the tube suffers a kink. The shock is assumed to move with velocity $\mathbf{U}_{SH}$ in direction $\hat{\mathbf{e}}_{SH}$ in the laboratory frame, where generally $\hat{\mathbf{e}}_{SH} \neq \hat{\mathbf{e}}_n$.

As the shock occurs only inside, it can not move away from the tube. We thus have for the perpendicular ($\perp$) components

$$(\mathbf{U}_{SH} \cdot \hat{\mathbf{e}}_{\perp 1})\, \hat{\mathbf{e}}_{\perp 1} = \mathbf{u}_{\perp 1} \quad , \quad (\mathbf{U}_{SH} \cdot \hat{\mathbf{e}}_{\perp 2})\, \hat{\mathbf{e}}_{\perp 2} = \mathbf{u}_{\perp 2} \quad . \tag{1}$$

In the frame, comoving with the shock we have the flow velocities

$$\mathbf{v}_1 = \mathbf{u}_1 - \mathbf{U}_{SH} \quad , \quad \mathbf{v}_2 = \mathbf{u}_2 - \mathbf{U}_{SH} \quad , \tag{2}$$

from which with Eqs. (1) we find

$$\mathbf{v}_{\perp 1} = \mathbf{v}_{\perp 2} = 0 \quad . \tag{3}$$

We now make the *thin flux tube approximation* (see e.g. Herbold et al. 1985, Ulmschneider et al. 1991), assume horizontal pressure balance and magnetic flux conservation

$$p_1 + \frac{B_1^2}{8\pi} = p_2 + \frac{B_2^2}{8\pi} = p_e \quad , \tag{4}$$

$$B_1 A_1 = B_2 A_2 = \text{constant} \quad . \tag{5}$$

2. Oblique shocks

Generally the four unit vectors $\hat{\mathbf{l}}_1, \hat{\mathbf{l}}_2, \hat{\mathbf{e}}_{SH}, \hat{\mathbf{e}}_n$ have different directions. We have derived the conservation laws for the thin tube with an oblique shock and find that these equations lead to a symmetrical kink at the shock, $\varphi_1 = \varphi_2$, or

$$\hat{\mathbf{e}}_n = \frac{1}{2}\left(\hat{\mathbf{l}}_1 + \hat{\mathbf{l}}_2\right) \quad . \tag{6}$$

List of unknowns (where $U_{l1SH} = \mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_1$, and a, b denote the two $\perp$ components):

$$u_{\perp a1},\ u_{\perp b1},\ u_{l1},\ v_{\perp a1},\ v_{\perp b1},\ v_{l1},\ A_1,\ B_1,\ \rho_1,\ p_1 \quad , \tag{7}$$

$$u_{\perp a2},\ u_{\perp b2},\ u_{l2},\ v_{\perp a2},\ v_{\perp b2},\ v_{l2},\ A_2,\ B_2,\ \rho_2,\ p_2 \quad , \tag{8}$$

$$U_{l1SH}, l_{x1}, l_{y1}, l_{z1}, l_{x2}, l_{y2}, l_{z2}, l_{xSH}, l_{ySH}, l_{zSH}, l_{xn}, l_{yn}, l_{zn} \,. \tag{9}$$

List of equations: We eliminate z-direction cosines using $l_{x1}^2 + l_{y1}^2 + l_{z1}^2 = 1$, $l_{x2}^2 + l_{y2}^2 + l_{z2}^2 = 1$, $l_{xSH}^2 + l_{ySH}^2 + l_{zSH}^2 = 1$ and replace $\hat{\mathbf{e}}_n$ by Eq. (6). Eqs. (4) and (5) allow to eliminate the four variables B_1, B_2, A_1, A_2 in favour of p_1, p_2. From Eqs. (3), $v_{\perp a1} = v_{\perp b1} = v_{\perp a2} = v_{\perp b2} = 0$, and Eqs. (2) give $u_{\perp a1} = u_{\perp a2} = U_{\perp aSH}$, $u_{\perp b1} = u_{\perp b2} = U_{\perp bSH}$, where $U_{\perp aSH}$, $U_{\perp bSH}$ can be written in terms of U_{l1SH} and l_{x1}, l_{y1}, l_{x2}, l_{y2}, l_{xSH}, l_{ySH}. With Eqs. (2), v_{l1}, v_{l2} can be eliminated in favour of u_{l1}, u_{l2} as well as

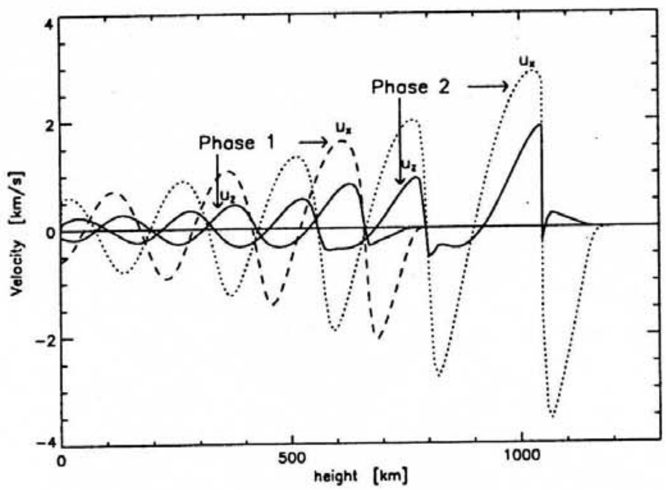

Figure 1: Vertical u_z and horizontal u_x velocity versus height for an mhd tube wave at phase 1 and with an oblique shock at a later phase 2.

U_{l1SH} and l_{x1}, l_{y1}, l_{x2}, l_{y2}, l_{xSH}, l_{ySH}. It is seen that only 7 longitudinal unknowns

$$u_{l1},\ \rho_1,\ p_1,\ u_{l2},\ \rho_2,\ p_2,\ U_{l1SH} \quad , \tag{10}$$

together with 6 transverse unknowns

$$l_{xSH},\ l_{ySH},\ l_{x1},\ l_{y1},\ l_{x2},\ l_{y2} \quad , \tag{11}$$

remain. The 7 longitudinal unknowns can be treated similarly as in Herbold et al. (1985), the 6 transverse unknowns (with 2 transverse characteristics in front and one behind the shock) as in Ulmschneider et al. (1991).

Using a time-dependent code we have followed the development of a wave introduced by longitudinal and transverse perturbations. Fig. 1 shows two subsequent wave phases. Strong transverse and longitudinal shocks develop. A first result is that despite of the different longitudinal and transverse wave speeds a shock disturbance with a common speed develops. This is a very interesting non-linear property which will be of great significance for the heating and momentum deposition of such shocks initiated by mode-coupling and purely horizontal perturbations.

References

Herbold, G., Ulmschneider, P., Spruit, H.C., Rosner, R.: 1985, *Astron. Astrophys.* **145**, 157.

Ulmschneider, P., Zähringer, K., Musielak, Z.: 1991, *Astron. Astrophys.* **241**, 625.