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Propagation of nonlinear longitudinal-transverse waves along magnetic flux tubes in the solar atmosphere

II. The treatment of shocks

Y.D. Zhugzhda[1,2], **V. Bromm**[1], **and P. Ulmschneider**[1]

[1] Institut für Theoretische Astrophysik der Universität Heidelberg, Im Neuenheimer Feld 561, D-69120 Heidelberg, Germany
[2] Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences, Troitsk, Moscow Region 142092, Russia

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Abstract. Equations were derived permitting the treatment of shocks which occur in longitudinal-transverse waves propagating in thin magnetic flux tubes. Such waves usually lead to kinks which may grow into oblique shocks. The kink angles between the shock and the tube directions are found to be symmetrical with respect to the shock. Purely longitudinal shocks occur only in special cases of high symmetry or of well defined propagation conditions. Wave calculations were performed which show the formation of oblique shocks. We also discuss recent suggestions that the basic equations for the treatment of longitudinal-transverse waves are incomplete.

Key words: shock waves – magnetohydrodynamics – Sun: magnetic fields

1. Introduction

A powerful approach to study the behaviour of waves in intense magnetic flux tubes observed on the sun is the *thin tube approximation*, which considers the tube as a one-dimensional sequence of mass elements, where the physical variables do not change much across the tube from their values on the tube axis. In this approach one usually assumes further that the flux tube is embedded in an otherwise nonmagnetic, time-independent solar atmosphere and that the motion of the tube, aside of displacing matter, does not perturb much the outside atmosphere. With these assumptions a formidable time-dependent problem, to treat magnetic tubes in three dimensions in an external atmosphere which varies in all three spatial directions, can be reduced to a more easily manageable one-dimensional problem. The properties of waves in thin flux tubes and the use of the thin tube approximation have been extensively discussed both in time-independent (e.g. Defouw 1976; Roberts & Webb 1978, 1979; Wilson 1979; Parker 1979; Wentzel 1979; Spruit 1981, 1982; Rae & Roberts 1982; Edwin & Roberts 1983) and time-dependent investigations (e.g. Herbold et al. 1985; Molotovshchikov & Ruderman 1987; Ferriz-Mas et al. 1989; Ulmschneider et al. 1991, henceforth called Paper I).

Our present work is an extension of Paper I. In that paper, following Spruit (1981), we listed the magnetohydrodynamic equations in the thin tube approximation and solved them using the method of characteristics in order to study the time-dependent propagation of longitudinal-transverse waves along solar magnetic flux tubes. Unlike to the situation in many finite difference methods, the method of characteristics treats shocks as discontinuities, which must be handled differently from regular interior points, as they represent true internal boundaries. Failure to treat shocks adequately will cause the numerical procedure to break down. It is thus a severe handicap of the procedure discussed in Paper I, that its methods are applicable only to cases where shocks do not occur.

In a previous paper (Herbold et al. 1985) the simpler case of pure longitudinal tube waves propagating along a vertical flux tube was discussed, together with the treatment of longitudinal shocks. As these shocks occur only under very special symmetry or propagation conditions, it is essential to look for a more general treatment of shocks allowing for kinks. The aim of this paper is thus to extend both the Herbold et al. treatment and Paper I, to derive the equations which govern the oblique shocks in the more general longitudinal-transverse wave case, and to study the shock formation in these waves. It has to be pointed out that the shocks which we discuss here are not yet the fully general oblique switch-on shocks, which occur when torsional motions (see e.g. Ferriz-Mas et al. 1989) are also considered. To include torsion (twist) is momentarily beyond the scope of this work.

Even in our present longitudinal-transverse case the discussion of the completeness of the set of equations may not be finished. Choudhuri (1990) and later Cheng (1992) correctly found that some centrifugal and Coriolis force terms are missing in the equations used by Spruit (1981) and in Paper I. These force terms arise when longitudinal flows are constrained to move along a curved and a swaying tube, respectively. Fluid flow along a swaying tube can act like the motion in a rotating system with a horizontally directed rotation axis. However, as the longitudinal flows in the tube are usually small compared to the sound or Alfvén speeds, the question is whether these missing terms are important in the framework of our approximate treatment. The paper is organized as follows: Section 2 describes the geometry and gives some basic relations. Section 3 derives the oblique shock case and discusses the situations where purely longitudinal shocks occur. Section 4 presents wave calculations with the development of shocks and discusses the importance of the mentioned missing terms. Section 5 gives our conclusions.

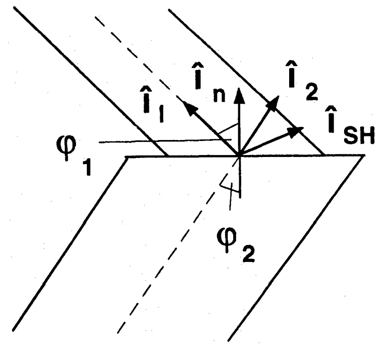

Fig. 1. Tube geometry and unit vectors at an oblique shock, the shock propagates towards the top of the figure

2. Geometry, velocities

In a stellar atmosphere consider a thin, roughly vertically directed magnetic flux tube with the field strength **B** along the tube, embedded in a non-magnetic external medium. At some height z_S we assume an oblique shock which is supposed to propagate towards greater height (see Fig. 1). At height $z < z_S$, in the region immediately behind the shock, the tube points in the direction $\hat{\mathbf{l}}_2$ and has a cross-sectional area A_2, while at height $z > z_S$ immediately in front of the shock, it points in the direction $\hat{\mathbf{l}}_1$ and has the cross-sectional area A_1. Here ^ denotes a unit vector.

The physical state inside the tube behind the shock is given by the gas velocity $\mathbf{u}_2$, the density ρ_2, the gas pressure p_2 and the magnetic field strength B_2. $\mathbf{u}_2$ is the three-dimensional velocity in the laboratory frame and is not necessarily directed along the tube. The gas pressure external to the tube is p_e. To make a one-dimensional treatment possible we assume that p_e depends only on height but not on time (see Roberts & Webb 1979; Rae & Roberts 1982). In front of the shock the physical state in the tube is given by $A_1, \mathbf{u}_1, \rho_1, p_1$ and B_1. Let us split the velocities $\mathbf{u}_1$ and $\mathbf{u}_2$ into the perpendicular (transverse) and longitudinal components $\mathbf{u}_{p1}$, $\mathbf{u}_{p2}$ and $\mathbf{u}_{l1}$, $\mathbf{u}_{l2}$, with respect to the tube.

The normal of the shock $\hat{\mathbf{l}}_n$ is assumed to be inclined by an angle φ_2 against the tube axis $\hat{\mathbf{l}}_2$ behind the shock and by an angle φ_1 against $\hat{\mathbf{l}}_1$ in front of the shock, such that the tube suffers a kink by an angle $\psi = \varphi_1 + \varphi_2$ at the shock. Note that the angles φ_1, φ_2 can be positive or negative, and are assumed to be small (much less than 90°).

Let A_{n2} and A_{n1} be the back and front areas of the shock surface, respectively, and $\mathbf{U}_{SH} = U_{SH}\hat{\mathbf{l}}_{SH}$ the velocity of the shock in the laboratory frame. Figure 2, somewhat exaggerated, shows a very thin flux tube (represented by a solid line) with a kink shock at some time t, and at a somewhat later time $t + \Delta t$. Note that in general $\hat{\mathbf{l}}_{SH} \neq \hat{\mathbf{l}}_1, \hat{\mathbf{l}}_2$. As the shock cannot move away from the tube we must have that

$$\left(\mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_{p1}\right) \hat{\mathbf{l}}_{p1} = \left(\mathbf{u}_1 \cdot \hat{\mathbf{l}}_{p1}\right) \hat{\mathbf{l}}_{p1} = \mathbf{u}_{p1} \quad ,$$

$$\left(\mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_{p2}\right) \hat{\mathbf{l}}_{p2} = \left(\mathbf{u}_2 \cdot \hat{\mathbf{l}}_{p2}\right) \hat{\mathbf{l}}_{p2} = \mathbf{u}_{p2} \quad , \tag{2.1}$$

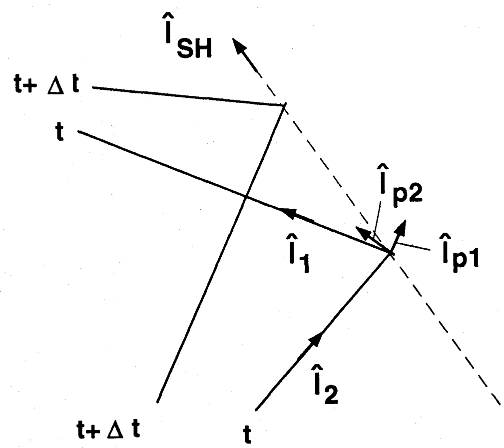

Fig. 2. Thin flux tube (solid) with an oblique shock at time t and at a later time $t + \Delta t$. The shock path (dashed) is not along the tube. Also shown are unit vectors mentioned in the text

where $\hat{\mathbf{l}}_{p1} \perp \hat{\mathbf{l}}_1$ and $\hat{\mathbf{l}}_{p2} \perp \hat{\mathbf{l}}_2$ are unit vectors pointing into the swaying direction of the tube in front and behind the shock (see Fig. 2). We now want to go into the frame which comoves with the shock, that is, moves with the velocity $\mathbf{U}_{SH}$ in direction $\hat{\mathbf{l}}_{SH}$. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities in front and behind the shock, respectively, in the comoving frame. We then have

$$\mathbf{v}_1 = \mathbf{u}_1 - \mathbf{U}_{SH} \quad , \quad \mathbf{v}_2 = \mathbf{u}_2 - \mathbf{U}_{SH} \quad . \tag{2.2}$$

The dot product of Eqs. (2.2) with the vectors $\hat{\mathbf{l}}_{p1}$ and $\hat{\mathbf{l}}_{p2}$, using Eqs. (2.1) gives

$$\mathbf{v}_{p1} = \mathbf{v}_{p2} = 0 \quad , \tag{2.3}$$

that is, only the longitudinal components $\mathbf{v}_{l1}$, $\mathbf{v}_{l2}$ remain in the comoving frame. The longitudinal components of Eqs. (2.2) can be written

$$v_{l1} = u_{l1} - \mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_1 \quad , \quad v_{l2} = u_{l2} - \mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_2 \quad . \tag{2.4}$$

3. Oblique shocks

3.1. Jump conditions at the shock

From Eq. (2.3) it is clear that, because the magnetic flux as well as the mass and energy fluxes must be conserved across the shock, we have very similar jump conditions as those derived by Herbold et al. (1985), namely

$$B_1 A_1 = B_2 A_2 = \Phi \quad , \tag{3.1}$$

$$\rho_1 v_{l1} A_1 = \rho_2 v_{l2} A_2 = j \quad , \tag{3.2}$$

$$\frac{1}{2} v_{l1}^2 + W_1 = \frac{1}{2} v_{l2}^2 + W_2 \quad , \tag{3.3}$$

where W is the specific enthalpy. As the momentum conservation involves the motion at a kink and leads to a symmetry condition for the kink angle, we must take particular care and rederive this jump condition from the basic equations. The equation of motion can be written

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right)$$

$$= -\nabla p - \nabla \frac{B^2}{8\pi} + \frac{1}{4\pi} (\mathbf{B} \cdot \nabla) \mathbf{B} - \rho g \hat{\mathbf{e}}_z \quad , \tag{3.4}$$

where g is the gravitational acceleration and $\hat{\mathbf{e}}_z$ the unit vector in the outward vertical direction. From Eq. (3.4) for thin tubes one can derive

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho v_n \frac{\partial \mathbf{v}}{\partial n} = -\hat{\mathbf{n}} \frac{\partial}{\partial n} \left(p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi} B_n \frac{\partial \mathbf{B}}{\partial n} - \rho g \hat{\mathbf{e}}_z \quad . \tag{3.5}$$

Here n is the direction normal to the shock surface and s is some direction along the shock surface. This equation is similar to those derived for thin tubes by authors cited in the introduction, except that here it is written for the normal direction of the shock surface. In accordance with the thin flux tube approximation that there is little variation across the tube, we assume that there is little variation along directions within the shock surface ($\partial/\partial s \approx 0$). This is assured by assuming that the tube is quite thin and that the angles between $\hat{\mathbf{l}}_1$ and $\hat{\mathbf{l}}_n$, between $\hat{\mathbf{l}}_n$ and $\hat{\mathbf{l}}_2$ (cf. Fig. 1) as well as between $\hat{\mathbf{l}}_n$ and $\hat{\mathbf{e}}_z$ are not too large, say much less than 90^0.

Taking the n-component of Eq. (3.5) and multiplying by A_n gives

$$\rho A_n \frac{\partial v_n}{\partial t} + \rho v_n A_n \frac{\partial v_n}{\partial n} =$$

$$-A_n \frac{\partial}{\partial n} \left(p + \frac{B^2}{8\pi} - \frac{B_n^2}{8\pi} \right) - \rho A_n g \hat{\mathbf{n}} \cdot \hat{\mathbf{e}}_z \quad . \tag{3.6}$$

Adding the continuity equation

$$\left(\frac{\partial \rho A_n}{\partial t} \right)_n + \left(\frac{\partial \rho v_n A_n}{\partial n} \right)_t = 0 \quad , \tag{3.7}$$

multiplied by v_n we have

$$\frac{\partial}{\partial t} (\rho v_n A_n) + \frac{\partial}{\partial n} (\rho v_n^2 A_n) =$$

$$-A_n \frac{\partial}{\partial n} \left(p + \frac{B_s^2}{8\pi} \right) - \rho A_n g_n . \tag{3.8}$$

With $g_n = g \hat{\mathbf{n}} \cdot \hat{\mathbf{e}}_z$ and bringing A_n under the differential operator, similarly as in Herbold et al. (1985, Eqs. 24 to 27), we obtain

$$\frac{\partial}{\partial t} (\rho v_n A_n) + \frac{\partial}{\partial n} \left[A_n \left(\rho v_n^2 - 2p_e + 2p + \frac{B_s^2}{4\pi} \right) \right] +$$

$$A_n \frac{dp_e}{dn} + \rho A_n g_n = 0 \quad , \tag{3.9}$$

from which we get the jump relations

$$A_{n1} \left(\rho v_{n1}^2 - 2p_e + 2p_1 + \frac{B_{s1}^2}{4\pi} \right) =$$

$$A_{n2} \left(\rho v_{n2}^2 - 2p_e + 2p_2 + \frac{B_{s2}^2}{4\pi} \right) \quad , \tag{3.10}$$

which, using horizontal pressure balance,

$$p_1 + \frac{B_1^2}{8\pi} = p_2 + \frac{B_2^2}{8\pi} = p_e \quad , \tag{3.11}$$

can be written

$$A_{n1} \left(\rho v_{n1}^2 - \frac{1}{4\pi} B_{n1}^2 \right) = A_{n2} \left(\rho v_{n2}^2 - \frac{1}{4\pi} B_{n2}^2 \right) \quad . \tag{3.12}$$

Because of the kink we must also consider the s-component of Eq. (3.5). Multiplying by A_n we get for this component

$$\rho A_n \frac{\partial v_s}{\partial t} + \rho v_n A_n \frac{\partial v_s}{\partial n} = A_n \frac{B_n}{4\pi} \frac{\partial B_s}{\partial n} - \rho A_n g \hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_z \quad , \tag{3.13}$$

from which, by adding the Eq. (3.7) multiplied by v_s we have with $g_s = g \hat{\mathbf{s}} \cdot \hat{\mathbf{e}}_z$ and using magnetic flux conservation

$$\frac{\partial}{\partial t} (\rho v_s A_n) + \frac{\partial}{\partial n} \left[A_n \left(\rho v_n v_s - \frac{1}{4\pi} B_n B_s \right) \right] + \rho A_n g_s = 0 . \tag{3.14}$$

This gives an additional jump condition

$$A_{n1} \left(\rho_1 v_{n1} v_{s1} - \frac{1}{4\pi} B_{n1} B_{s1} \right) =$$

$$A_{n2} \left(\rho_2 v_{n2} v_{s2} - \frac{1}{4\pi} B_{n2} B_{s2} \right) \quad . \tag{3.15}$$

This relation is valid independently for both transverse components of the magnetic field.

3.2. Symmetry condition for the oblique shock

We now want to show that Eqs. (3.12) and (3.15) lead to a symmetrical kink at the shock. Writing these equations in terms of mass flux j and magnetic flux Φ we have

$$j\, v_{n1} - \frac{\Phi}{4\pi} B_{n1} = j\, v_{n2} - \frac{\Phi}{4\pi} B_{n2} \quad , \tag{3.16}$$

$$j\, v_{s1} - \frac{\Phi}{4\pi} B_{s1} = j\, v_{s2} - \frac{\Phi}{4\pi} B_{s2} \quad , \tag{3.17}$$

We now write the n- and s- components in terms of the longitudinal components and have

$$B_{n1} = B_1 \cos\varphi_1 \quad , \quad B_{n2} = B_2 \cos\varphi_2 \quad , \tag{3.18}$$

$$v_{n1} = v_{l1} \cos\varphi_1 \quad , \quad v_{n2} = v_{l2} \cos\varphi_2 \quad , \tag{3.19}$$

$$B_{s1} = B_1 \sin\varphi_1 \quad , \quad B_{s2} = B_2 \sin\varphi_2 \quad , \tag{3.20}$$

$$v_{s1} = v_{l1} \sin\varphi_1 \quad , \quad v_{s2} = v_{l2} \sin\varphi_2 \quad , \tag{3.21}$$

from which we find

$$\cos\varphi_1 \left(j v_{l1} - \frac{\Phi}{4\pi} B_1 \right) = \cos\varphi_2 \left(j v_{l2} - \frac{\Phi}{4\pi} B_2 \right) \quad , \tag{3.21}$$

$$\sin\varphi_1 \left(j v_{l1} - \frac{\Phi}{4\pi} B_1 \right) = \sin\varphi_2 \left(j v_{l2} - \frac{\Phi}{4\pi} B_2 \right) \quad . \tag{3.22}$$

This implies

$$\frac{\cos\varphi_1}{\cos\varphi_2} = \frac{\sin\varphi_1}{\sin\varphi_2} \quad , \tag{3.23}$$

which is only true if

$$\varphi_1 = \varphi_2 \quad . \tag{3.24}$$

With

$$A_{n1} = \frac{A_1}{\cos\varphi_1} \quad , \quad A_{n2} = \frac{A_2}{\cos\varphi_2} \quad , \tag{3.25}$$

we obtain finally

$$A_1 \left(\rho_1 v_{l1}^2 - \frac{B_1^2}{4\pi} \right) = A_2 \left(\rho_2 v_{l2}^2 - \frac{B_2^2}{4\pi} \right) \quad , \tag{3.26}$$

which is very similar to the Herbold et al. (1985) result and which from Eq. (2.3) might have already been expected. However, this result is due precisely to the symmetry condition of Eq. (3.24) which is not trivial. For oblique shocks we thus have the magnetic flux condition (Eq. 3.1), the three Hugoniot-type jump conditions (Eqs. 3.2, 3.3, 3.26) and the kink angle symmetry condition (Eq. 3.24).

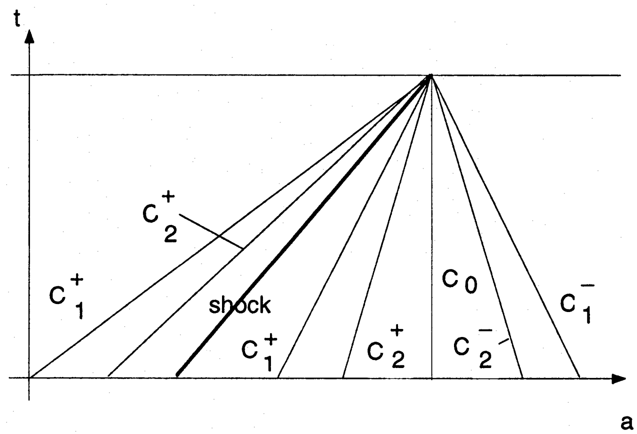

Fig. 3. Characteristics in the time-space plane for the oblique shock case

3.3. Set of equations for a time-dependent treatment

For our time-dependent treatment it is not sufficient to derive the jump conditions at the shock, but we must make sure that we are able to completely predict the behaviour of all physical quantities before and behind the shock together with the motion of the shock. For this reason we have to carefully compare the number of unknowns with the number of equations in order to determine whether we have enough relations. Let us denote the two directions perpendicular to the tube by a, b and as usual the longitudinal direction by l.

List of unknowns:
In front of the shock we have the unknowns

$$u_{a1},\ u_{b1},\ u_{l1},\ v_{a1},\ v_{b1},\ v_{l1},\ A_1,\ B_1,\ \rho_1,\ p_1 \quad , \tag{3.27}$$

behind the shock there are the unknowns

$$u_{a2},\ u_{b2},\ u_{l2},\ v_{a2},\ v_{b2},\ v_{l2},\ A_2,\ B_2,\ \rho_2,\ p_2 \quad , \tag{3.28}$$

and in addition

$$U_{l1SH},\ l_{x1},\ l_{y1},\ l_{z1},\ l_{x2},\ l_{y2},\ l_{z2},$$

$$l_{xSH},\ l_{ySH},\ l_{zSH},\ l_{xn},\ l_{yn},\ l_{zn}. \tag{3.29}$$

Here we have assumed that all other thermodynamic variables can be computed using ρ, p. We further assume that l_{x1}, l_{y1}, l_{z1}, etc. are the direction cosines with respect to the x, y, z-axis of the four unit vectors $\hat{\mathbf{l}}_1$, $\hat{\mathbf{l}}_2$, $\hat{\mathbf{l}}_{SH}$ and $\hat{\mathbf{l}}_n$. Furthermore $U_{l1SH} = \mathbf{U}_{SH} \cdot \hat{\mathbf{l}}_1$.

List of equations:
For the direction cosines of the four unit vectors we have the conditions

$$l_{x1}^2 + l_{y1}^2 + l_{z1}^2 = 1 \quad , \quad \text{etc.} \tag{3.30}$$

which can be used to eliminate l_{z1}, l_{z2}, l_{zSH}, l_{zn}. Because of the symmetry condition (3.24) we have

$$\hat{\mathbf{l}}_n = \frac{\hat{\mathbf{l}}_1 + \hat{\mathbf{l}}_2}{\left|\hat{\mathbf{l}}_1 + \hat{\mathbf{l}}_2\right|} \quad , \tag{3.31}$$

which allows to compute l_{xn}, l_{yn} as well. The four Eqs. (3.1) and (3.11) permit to eliminate the four variables B_1, B_2, A_1, A_2 in favour of p_1, p_2 and the four relations (2.3) give $v_{a1} = v_{b1} = v_{a2} = v_{b2} = 0$. Eqs. (2.1) provide $U_{aSH} = u_{a1} = u_{a2}$, $U_{bSH} = u_{b1} = u_{b2}$ and give u_{a2}, u_{b2} in terms of U_{l1SH} and l_{x1}, l_{y1}, l_{x2}, l_{y2}, l_{xSH}, l_{ySH}. With Eqs. (2.4), v_{l1}, v_{l2} can be eliminated in favour of u_{l1}, u_{l2} as well as U_{l1SH} and l_{x1}, l_{y1}, l_{x2}, l_{y2}, l_{xSH}, l_{ySH}. It is seen that only 7 unknowns

$$u_{l1},\ \rho_1,\ p_1,\ u_{l2},\ \rho_2,\ p_2,\ U_{l1SH} \quad , \tag{3.32}$$

together with 6 unknowns

$$l_{xSH},\ l_{ySH},\ l_{x1},\ l_{y1},\ l_{x2},\ l_{y2} \quad , \tag{3.33}$$

remain. As the unknowns of the lists (3.32) and (3.33) belong to the C_1 and C_2 characteristics systems which are associated with the longitudinal and transverse wave modes, we call them longitudinal and transverse unknowns, respectively. The system of characteristics at an oblique shock is shown in Fig. 3. The 7 longitudinal unknowns can be treated similarly as in the Herbold et al. (1985) case: 4 variables are determined by using one relation each along the four (two C_1^+, one C_0, one C_1^-) characteristics and by 3 Hugoniot relations (Eqs. (3.2), (3.3), (3.26)). The case treated by Herbold et al. (1985) is reproduced after setting equal to zero all six values listed in Eq. (3.33). Note that in that case the characteristics C_2^+, C_2^- shown in Fig. 3 will be absent.

The 6 transverse unknowns are treated using two relations each (for the two transverse directions a, b) along the 3 (two C_2^+ and one C_2^-) characteristics as seen in Fig. 3. This shows that we have exactly the right number of equations to determine the number of unknowns. We conclude that indeed we have a necessary and sufficient set of equations for the computation of the time-dependent solution.

3.4. *The occurrence of purely longitudinal shocks*

In the previous chapter we have discussed the general oblique shock case, we now ask whether there are situations in which purely longitudinal shocks (shocks without kinks) similarly to the Herbold et al. (1985) case are possible. It is clear that the Herbold et al. case is very special due to its high symmetry. In this case a vertical tube is excited by a purely vertical piston motion. From considerations of symmetry one has

$$\hat{\mathbf{l}}_1 = \hat{\mathbf{l}}_2 = \hat{\mathbf{l}}_{SH} = \hat{\mathbf{l}}_n \quad , \quad \hat{\mathbf{l}}_{p1} = \hat{\mathbf{l}}_{p2} \quad , \tag{3.34}$$

that is, the 6 transverse unknowns in the list (3.33) all vanish and the entire transverse mode is excluded by symmetry.

Are there other situations where purely longitudinal shocks are possible? The condition for such a shock is that there is no kink,

$$\varphi_1 = \varphi_2 = 0 \quad . \tag{3.35}$$

A careful listing of the unknowns and the relations available for this case shows that one retains the 7 longitudinal unknowns of the list (3.32), but that one has only 4 transverse unknowns

$$u_{a1},\ u_{b1},\ l_x,\ l_y \quad , \tag{3.36}$$

which represent the transverse velocity in front of the shock and the direction of the tube. The 7 longitudinal unknowns are treated similarly as in the Herbold et al. or the general oblique shock cases. But with 6 relations along the three C_2 characteristics (see Fig. 3) the 4 transverse unknowns are overdetermined. Note that because the 4 transverse unknowns are those present at an interior point, they can be computed using only the C_2^+ and C_2^- characteristics in front of the shock. This behaviour is reminiscent of the case of Paper I.

Unfortunately, this picture is not consistent with the fact that the transverse tube speed

$$c_k = c_A \sqrt{\frac{\rho}{\rho + \rho_e}} \quad , \tag{3.37}$$

suffers a jump, due to the jump in c_A and ρ at the shock. This shows that *the assumption of $l_{x1} = l_{x2}$, $l_{y1} = l_{y2}$, that is $\varphi_1 = \varphi_2$, must be wrong in most situations* because a different transverse tube speed on both sides of the shock should lead to a kink in the tube.

There is, however, another possibility. Consider Fig. 3. In front of the shock we have 5 characteristics due to the two tube speeds c_T, c_k. Behind the shock there are two (C_1^+, C_2^+) characteristics under the assumption that here the disturbances for both tube speeds travel faster than the shock. Yet there might be the case in which the shock travels faster than the C_2^+ characteristic. In this case, the C_2^+ characteristic behind the shock is absent, and we have exactly the situation described above where the transverse unknowns behave like at an interior point.

This shows that purely longitudinal shocks occur in two situations: a first case, where the transverse mode is absent due to symmetry, which is the Herbold et al. (1985) case, and a second case, where both tube wave modes occur, but where the shock moves faster than the transverse tube speed, c_k, behind the shock.

4. Wave calculation

4.1. *Oblique shock formation*

We now show results obtained by implementing the above derived equations in the time-dependent numerical code described in Paper I, based on the method of characteristics. In our first example of a wave calculation with oblique shocks we assume an initially vertical, exponentially spreading magnetic flux tube of 1500 km height with a basal field strength of B=1500 G, similar to the tube described in Paper I. The tube is perturbed by a sinusoidal piston motion along the tube and simultaneously by a sinusoidal transverse motion to generate both longitudinal and transverse waves. The periods of both excitations were P=30 s. The longitudinal amplitude was 0.2 km/s, the transverse shaking

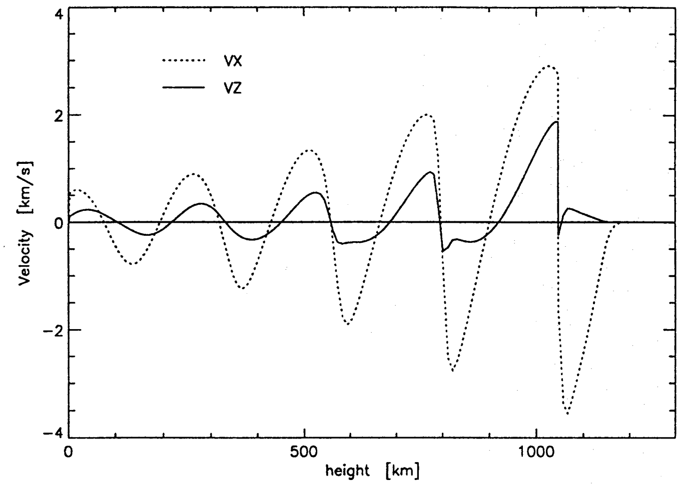

Fig. 4. Horizontal v_X and vertical v_Z velocity versus height for a longitudinal-transverse wave with an oblique shock. The wave is shown at time t = 144 s and was excited at the bottom of the magnetic flux tube by simultaneous longitudinal and transverse shaking

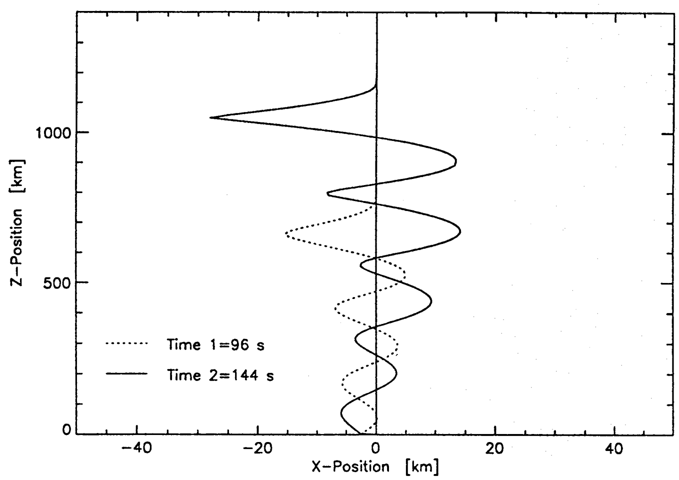

Fig. 5. Spatial display of the axis of the magnetic flux tube. For easy readability the horizontal scale has been greatly magnified. x is the horizontal, z the vertical distance. The wave is shown at time t = 144 s in the same phase as in Fig. 4 and at t = 96 s

amplitude 0.6 km/s. A phase shift of 45^0 was introduced with a leading longitudinal oscillation. The development of the wave was followed up to shock formation and beyond. Preliminary results for this wave calculation were reported by Zhugzhda et al. (1994).

Figure 4 shows the horizontal and vertical velocities, v_X and v_Z, of the wave at time t = 144 s after the start of the computation. At 1050 km height an oblique shock is seen while at 800 km a shock is about to form. The bumps in v_Z at 850 and 1100 km are due to the nonlinear coupling from the transverse oscillations which generates longitudinal perturbations of twice the frequency of the transverse wave. For more details on such a nonlinear coupling see Paper I. The geometrical shape of the tube axis in the xz-plane is shown in Fig. 5. Note the different

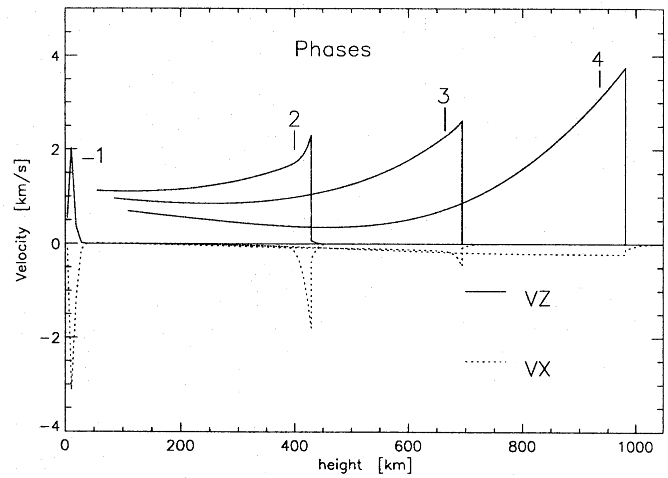

Fig. 6. Horizontal v_X and vertical v_Z velocity versus height for a longitudinal-transverse wave with an oblique shock. The wave is shown at times t = 3, 53, 82 and 110 s and was excited at the bottom of the magnetic flux tube by a pure transverse pulse of amplitude 5 km/s lasting for 1 s

scales of the vertical and horizontal distances. The shock at 1050 km height is associated with a kink by an angle of $\psi = \varphi_1 + \varphi_2 = 35^0$. Figure 5 shows that the wave at 800 km height is on its way to develop a kink shock as well. To show the development of the oblique shock out of a progressively sharpening kink, Fig. 5 displays the wave also at an earlier time t = 96 s. Here the physical meaning of the mathematical result $\varphi_1 = \varphi_2$ becomes clear, it is a symmetry condition, where the ever sharpening bend in the tube cracks in such a way that the shock lies in the symmetry axis.

In the present and other wave calculations we almost always find that the waves generate a common shock, that is, both the longitudinal and transverse waves form shocks together, and the shocks propagate with a common speed. This is surprising in view of the different propagation speeds, c_T and c_k, of the longitudinal and transverse waves and indicates the strong nonlinear mode-coupling introduced by the shock.

A second wave calculation is shown in Fig. 6. Here the same tube as above is excited by a pure transverse pulse with amplitude 5 km/s and 1 s duration. Events of this type and magnitude have been observed on the Sun by Muller et al. (1994) as well as Title (1994, private communication). The time- development of the horizontal and vertical velocity, v_X and v_Z, of this pulse is shown in Fig. 6 at times 3, 53, 82 and 110 s. It is seen that strong mode-coupling converts a large amount of energy into a longitudinal wave pulse which completely dominates the wave calculation. A very likely outcome of such a transverse pulse excitation is the formation of spicules (see Cheng 1992a, 1992b).

4.2. Missing terms in the basic equations

In the present wave calculations we did not yet implement the terms which Choudhuri (1990) and Cheng (1992) correctly claimed to be missing in the transverse momentum equation (Eq. 24 of Paper I), called Spruit's equation by Cheng (1992).

However, estimates of the importance of these terms for wave calculations of the type discussed in Paper I showed that although the correction term for the centrifugal force was fairly small, the correction term for the Coriolis force was quite important. It is thus definitely necessary to include these correction terms in future wave calculations, particularly in cases where considerable longitudinal flows occur in the tube. It should be noted, however, that the above mentioned missing terms do not change the main results of the present paper, namely our derived equations to handle shocks. This is because these equations represent the conservation of mass, momentum and energy in a tube together with a symmetry condition for the kink angle.

5. Conclusions

1. We have derived Hugoniot-type jump conditions which are valid for the treatment of oblique shocks in thin magnetic flux tubes. It was found that the kink angles between the shock and the tube directions are symmetrical with respect to the shock.
2. Time-dependent wave computations using these equations show that, despite of different propagation speeds of the longitudinal and transverse tube modes, a common shock develops which propagates with a common shock speed. This indicates that the shock introduces severe nonlinear mode-coupling which should be very important for the mechanical energy transport as well as for the heating and momentum transfer (wind generation) in magnetic flux tubes.
3. Purely longitudinal shocks (without a kink) develop not only in the trivial case where symmetry conditions suppress the transverse mode, but also when special propagation conditions apply.
4. The centrifugal and Coriolis terms which Choudhuri (1990) and Cheng (1992) correctly found to be missing in the equations which govern longitudinal-transverse waves in thin magnetic flux tubes depend on the magnitude of longitudinal flows introduced in the tube. For typical wave calculations the centrifugal term was relatively small but the Coriolis term was found to be quite important. It thus is necessary to include these missing terms in future wave calculations.

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