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FAST METHOD FOR CALCULATING CHROMOSPHERIC Ca II AND Mg II RADIATIVE LOSSES

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ABSTRACT

A fast and reasonably accurate method for calculating the total radiative losses by Ca II and Mg II ions for time-dependent chromospheric wave calculations has been developed. The method is based on a two-level atom procedure with pseudo–partial frequency redistribution (pseudo-PRD). The speed of the method is due to scaling of the total losses from single-line results. Acceleration of computation speeds by factors of roughly 10^2–10^3 can be achieved. The method is tested against the results from a modified version of the multilevel atom code MULTI.

Subject headings: methods: numerical — radiative transfer — stars: chromospheres

1. INTRODUCTION

In the process of constructing theoretical models of stellar chromospheres based on wave heating mechanisms, the time-dependent energy balance between the wave dissipation due to shocks and the emitted radiative losses has to be calculated many times at each height in a stellar atmosphere. From semiempirical solar modeling we know that the major sources of these chromospheric radiative losses are the H^- and hydrogen continua, as well as the hydrogen, Ca II, Mg II, and Fe II lines (Vernazza et al. 1981; Anderson & Athay 1989). It is necessary that these time-dependent radiative losses must be computed simultaneously with the calculation of energy dissipation by acoustic and magnetic waves and with the fully consistent and time-dependent computation of the hydrogen ionization (e.g., Carlsson & Stein 1997; Ulmschneider & Musielak 2003; Rammacher & Ulmschneider 2003).

Since the treatment of radiative losses requires a lot of computation time, several simplifications have been made to reduce the amount of computational labor. To calculate the total radiation losses from the Ca II and Mg II ions, models for nonmagnetic and magnetic regions have been constructed by computing radiative losses only in the single Ca II K and Mg II k lines and then scaling up these results by correction factors to account for the full radiative losses (e.g., Cuntz et al. 1999; Fawzy et al. 2002a, 2002b; Rammacher & Ulmschneider 2003). These correction factors were originally obtained by using the results of Vernazza et al. (1981) for their semiempirical solar atmosphere model C. For this model, Vernazza et al. (1981) computed the various radiative loss contributions of lines and continua by using a multilevel radiative transfer code called PANDORA (Avrett & Loeser 1992) based on the equivalent two-level atom method. A similar multilevel radiation code called MULTI was developed by Carlsson (1992, 1995) based on the complete linearization method.

In principle, the time-dependent energy balance in chromospheric wave calculations should be performed by using radiative loss codes such as PANDORA or MULTI for each ion. However, because of the slow convergence of these codes and due to the fact that the convergence properties can vary vastly for subsequent time steps, the mentioned single-line method is much more attractive since it has no convergence problems and is by factors of the order of 1000 faster. A drawback of that method so far was that reliable correction factors to scale the single-line losses up to the total losses were missing. It is the purpose of the present paper to put this single-line method on a more reliable basis and to evaluate correction factors from a direct comparison of single-line losses with total losses computed using MULTI.

Another problem in the computation of chromospheric radiation losses is that the resonance lines of Ca II and Mg II must be calculated by taking partial redistribution (PRD) into account. Calculations based on complete redistribution (CRD) are much faster, by factors of several thousand, but they yield line energy losses that are often wrong by orders of magnitude (Ulmschneider 1994). This is due to the fact that in realistic situations CRD occurs only over the line core of roughly ± 3 Doppler widths while the wings are formed by scattering. Line wings therefore do not contribute to the heating or cooling of the local gas element. However, as shown, e.g., by Hünerth & Ulmschneider (1995) and others, the treatment of PRD for a line can be approximated by performing a calculation using CRD while decreasing artificially the importance of the line wings by multiplying the damping parameter a in the Voigt function with factors of $f_{\rm psPRD} = 1/100$ or $1/1000$. This procedure is called pseudo–partial frequency redistribution (pseudo-PRD), and it accelerates the computation of line radiation losses by factors greater than 1000. Pseudo-PRD, therefore, is presently used in most of the chromospheric wave calculations (Carlsson & Stein 1997; Cuntz et al. 1994; Rammacher & Ulmschneider 2003; but for recent developments of fast PRD codes see also Uitenbroek 2001, 2002).

The aim of this paper is to describe our single-line method applied to the Ca II and Mg II radiation losses and to deduce the relevant radiation loss correction factors. The description of our two-level atom method is given in § 2, and in § 3 a detailed comparison of the single-line radiative losses for individual wave phases is provided with a full multiline calculation performed using MULTI. Since the available version of MULTI (Carlsson 1995) cannot handle shock discontinuities, modifications of MULTI were required. The obtained results are presented and discussed in § 4, while § 5 gives our conclusions.

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2. TWO-LEVEL ATOM METHOD

We now briefly describe our two-level atom method, henceforth called RAD2L. The equation of radiative transfer (e.g., Mihalas 1978, p. 350) is given by

$$\mu \frac{\partial I_{\nu\mu}(z)}{\partial \tau} = \left(\varphi_{\nu\mu} + r\right)\left[I_{\nu\mu}(z) - S_{\nu\mu}(z)\right]. \quad (1)$$

Here τ is the line optical depth with $d\tau = -\chi_L(z)\,dz$, where z is the geometrical height, $I_{\nu\mu}$ is the specific monochromatic intensity of the radiation field, $S_{\nu\mu}$ is the total source function, μ represents the angle cosine between the direction of photons and the outward normal, and ν is frequency. A two-level atom with a background radiation in LTE is considered. In addition, a two-beam approximation is made with one angle point and $\mu = 1/\sqrt{3}$. The line emission and absorption profiles are calculated assuming CRD. The total extinction coefficient is written as

$$\chi_{\nu\mu}(z) = \chi_C(z) + \chi_L(z)\varphi_{\nu\mu}(z), \quad (2)$$

where χ_L and χ_C are frequency-independent line and continuum extinctions, respectively, $r(z) = \chi_C/\chi_L$ is the residual strength of the continuum, and $\varphi_{\nu\mu}$ is the absorption profile given by the Voigt function

$$\varphi_{\nu\mu} = \frac{1}{\sqrt{\pi}\Delta\nu_{\rm D}} H(a,\ v), \quad (3)$$

with

$$H(a,\ v) = \frac{a}{\pi}\int_{-\infty}^{+\infty} \frac{e^{-y^2}\,dy}{(v-y)^2 + a^2}. \quad (4)$$

Here the damping parameter a is given by

$$a = \frac{\Gamma}{4\pi\Delta\nu_{\rm D}} f_{\rm psPRD}, \quad (5)$$

where Γ is the damping constant and $\Delta\nu_{\rm D}$ is the Doppler width given by

$$\Delta\nu_{\rm D} = \frac{\nu_{12}}{c}\left[\frac{2kT(z)}{m} + v_{\rm turb}^2(z)\right]^{1/2}, \quad (6)$$

where ν_{12} is the frequency at line center, $v_{\rm turb}$ represents the turbulent velocity of the background medium, k is the Boltzmann constant, m is mass of the considered ion, and c is the speed of light. We follow Hünerth & Ulmschneider (1995) and consider a pseudo-PRD factor $f_{\rm psPRD} = 1/100$. The normalized frequency offset v in the Voigt function is given by

$$v = \frac{\Delta\nu}{\Delta\nu_{\rm D}} = \frac{\nu - \nu_{12}[1 - \mu u(z)/c]}{\Delta\nu_{\rm D}}, \quad (7)$$

where $u(z)$ is the gas velocity.

The total damping constant is $\Gamma = \Gamma_{\rm rad} + \Gamma_E$, with

$$\Gamma_E = \Gamma_{\rm vdW}\left[\frac{T(z)}{5000}\right]^{0.3} n_{\rm H}(z), \quad (8)$$

where $n_{\rm H}$ is the number density of neutral hydrogen. Note that in the present work $u(z)$, $T(z)$, $n_{\rm H}(z)$, the electron number density $n_e(z)$, and the ion number density $n_1(z)$ are all specified functions of height z in the solar atmosphere. The turbulent velocity is assumed to be $v_{\rm turb} = 0$. The data for the Ca II K and Mg II k lines such as $\Gamma_{\rm rad}$, $\Gamma_{\rm vdW}$, A_{21}, and Ω_{21} used in our calculations are given in Table 1 of Kalkofen et al. (1984).

The absorption profile satisfies the normalization condition

$$\int_{-1}^{1}\int_{-\infty}^{\infty} \varphi_{\nu\mu}\,d\nu\,d\mu = 1. \quad (9)$$

The background source function is assumed to be the Planck function at line center

$$B(z) = B[T(z)], \quad (10)$$

while the line source function S_L for the two-level atom model with CRD (Mihalas 1978, p. 337) is given by

$$S_L(z) = [1 - \epsilon(z)]\overline{J}(z) + \epsilon(z)B(z), \quad (11)$$

with

$$\epsilon(z) = \frac{\epsilon'(z)}{1 + \epsilon'(z)}, \quad (12)$$

and with the photon destruction probability given by

$$\epsilon'(z) = \frac{n_e(z)\Omega_{21}}{A_{21}}\left(1 - e^{-h\nu/kT(z)}\right). \quad (13)$$

Here $n_e\Omega_{21}$ is the collisional de-excitation rate, A_{21} is the spontaneous recombination rate, and h is the Planck constant.

With the total source function $S_{\nu\mu}(z)$ given by (Mihalas 1978, p. 350)

$$S_{\nu\mu}(z) = \frac{\varphi_{\nu\mu}(z)S_L(z) + r(z)B(z)}{\varphi_{\nu\mu}(z) + r(z)}, \quad (14)$$

the transfer equation (1) can be integrated and the line profile–averaged mean intensity $\overline{J}(z)$ given by

$$\overline{J}(z) = \tfrac{1}{2}\int_{-\infty}^{\infty} d\nu \int_{-1}^{1} d\mu\,\varphi_{\nu\mu}(z)I_{\nu\mu}(z) = \Lambda\left[S_{\nu\mu}(z')\right] \quad (15)$$

can be evaluated. While the background source function $B(z)$ is known, the problem is to rapidly solve equations (14), (15), and (11) for the line source function S_L. Iterating among these three equations (lambda iteration) is very slow. Therefore, a revised operator perturbation method is employed (Buchholz et al. 1994) that uses an easily invertible approximate lambda operator Λ^* and for which the line profile–averaged intensity of the lth iteration can be written in terms of the $(l-1)$th iteration as

$$\overline{J}_l(z) = \left[1 - (1-\epsilon)\Lambda^*\right]^{-1}\left[\overline{J}_{\rm FS}(z) - (1-\epsilon)\Lambda^*\overline{J}_{l-1}(z)\right], \quad (16)$$

where the formal solution $\overline{J}_{\rm FS}(z)$ is computed by taking $\overline{J}(z) = \overline{J}_{l-1}(z)$ in equation (11) and subsequently evaluating equations (14) and (15).

Note that the solution of equation (16) involves heights z and z' where at the shocks the distances $z - z'$ are zero. For this reason in our operator perturbation method (Buchholz et al. 1994) the transfer equation is integrated using a piecewise linear interpolation of the source function (Kalkofen & Ulmschneider

1984) where for an outgoing ray the specific intensities with height index i and angle-frequency index k can be written as

$$I_{ik}^{+} = a_{ik}^{+} I_{(i+1)k}^{+} + b_{ik}^{+} S_{ik} + c_{ik}^{+} S_{(i+1)k}, \tag{17}$$

with

$$a_{ik}^{+} = \begin{cases} \dfrac{2-\delta}{2+\delta}; \quad b_{ik}^{+} = c_{ik}^{+} = \dfrac{\delta}{2+\delta}, & \delta \le 1, \\ c_{ik}^{+} = \dfrac{1}{2\delta+1}; \quad b_{ik}^{+} = \dfrac{2\delta-1}{2\delta+1}, & \delta > 1, \end{cases} \tag{18}$$

where

$$\delta = \tau_{(i+1)k} - \tau_{ik}. \tag{19}$$

Here τ_{ik} denotes the optical depth at grid point i along the outgoing ray with index k. The superscript "+" denotes outgoing ($\mu > 0$) radiation. Similarly, we find for the ingoing ($\mu < 0$, superscript "−") intensities

$$I_{ik}^{-} = a_{ik}^{-} I_{(i-1)k}^{-} + b_{ik}^{-} S_{ik} + c_{ik}^{-} S_{(i-1)k}, \tag{20}$$

where the values a_{ik}^{-}, b_{ik}^{-}, c_{ik}^{-} are the same as the a_{ik}^{+}, b_{ik}^{+}, c_{ik}^{+} except that now

$$\delta = \tau_{ik} - \tau_{(i-1)k}. \tag{21}$$

The construction of the lambda and approximate lambda operators, Λ and Λ^{*}, using these solutions of the transfer equation, is discussed by Buchholz et al. (1994).

For the treatment of the Ca II K and Mg II k line emissions, we have selected 29 logarithmically spaced frequency points $\Delta\lambda_k$ to cover each line and have used two angle points $\mu = \pm 1/\sqrt{3}$. The maximum frequency offsets from line center, $\Delta\lambda_1$ and $\Delta\lambda_{29}$, were selected such that the line cores were well covered. Integrating the emergent line profiles $I^{+}(\Delta\lambda)$ over wavelength $\Delta\lambda$ in the line core (between ± 3 Å), the K and k line radiative energy fluxes $F = 2\pi\mu I^{+}$ are computed.

The net radiative cooling rate due to the line in the two-level approximation (Kalkofen & Ulmschneider 1984) can finally be written as

$$\Phi_R = h\nu_{12} n_1 B_{12} \epsilon \frac{B - S_L}{1 + c^2 S_L / \left(2h\nu_{12}^3\right)}, \tag{22}$$

where n_1 is the number density of the ground state and B_{12} is the Einstein coefficient. Since in our hydrodynamic wave calculations we use the entropy per gram, S, as a convenient thermodynamic variable that monitors the gain and loss of energy in chromospheric gas elements (Ulmschneider et al. 1977; Ulmschneider & Kalkofen 1978), we prefer to use the radiative damping function D (the rate of entropy increase per time) instead of the radiative cooling rate Φ_R. For the radiative damping function due to lines we therefore have

$$D = \left.\frac{dS}{dt}\right|_{\text{line}} = -\frac{\Phi_R}{\rho T}, \tag{23}$$

where ρ is the density.

3. MODIFICATIONS OF MULTI AND THE TEST OF RAD2L

In order to compare the intensities and radiative energy fluxes of the chromospheric emitters Ca II and Mg II computed with our two-level code RAD2L, the multilevel atom transfer code MULTI (Carlsson 1986, 1995) has been used. Three major changes had to be made to MULTI so that it could be used in our wave calculations.

First, our code uses the modified characteristics method where shocks are treated as discontinuities with zero geometrical and optical depth distances through the shock jump. For MULTI to be able to handle such zero optical depth grid distances, we use linearly interpolated source functions and employ a local (diagonal) operator based on the procedure originally developed by Kalkofen & Ulmschneider (1984). These modifications are described in equations (17)–(21).

Second, in our wave calculations (Rammacher & Ulmschneider 2003), for the purpose of computing the time-dependent ionization of hydrogen, as well as the several ionization stages of Ca and Mg, we solve the time-dependent rate equations. In the available version of MULTI (Carlsson 1995), however, only the statistical rate equations are solved where the finite relaxation times for the ionization equilibria are not taken into account. An unmodified MULTI would thus recompute the ionization equilibria of our wave computation in an incorrect manner, while for the bound-bound transitions with their very short relaxation times the use of the statistical rate equations is valid. The method used by MULTI, therefore, must be retained for bound-bound transitions, while for the bound-free transitions it has to be modified. We achieve this by adopting the ionization degrees from the wave calculation and force MULTI to retain these ionization degrees by artificially raising the ionization energies of the continuum levels by factors of between 4 and 8, which is sufficient to suppress effectively transitions into the continuum.

Third, the line emission fluxes are computed using pseudo-PRD; that is, we follow Hünerth & Ulmschneider (1995) and artificially decrease the damping parameter in the lines by a factor of 100, the same as in the wave code (see eq. [5]).

With these modifications of MULTI the next step was to test the validity and accuracy of our RAD2L code. Since MULTI allows us to freely choose the atomic model, we selected a two bound level plus continuum atomic model to simulate our two bound level RAD2L code. For comparison we prepared several snapshots of acoustic wave calculations in the solar atmosphere that include shocks. The wave calculations started from an initial hydrostatic radiative equilibrium solar atmosphere model in which by means of time-dependent piston motions at the lower boundary we excite acoustic waves. For this test we chose a monochromatic acoustic wave of $P = 20$ s and an energy flux $F_M = 1.7 \times 10^8$ ergs cm^{-2} s^{-1}.

After reaching a dynamically stable state of the atmosphere, we extracted four consecutive time steps (wave phases) from the wave calculation roughly $\Delta t = P/4$ apart that cover the typical time variation of physical parameters in the atmosphere. These snapshots were used as input for RAD2L and MULTI. Figure 1, for the Ca II K line, shows the temperature T and damping function D of the first of these snapshots at time $t = 1483.3$ s for the height interval $z = 500$–1300 km. The damping functions computed with RAD2L are shown by a solid line, and those with MULTI by a dotted line.

Clearly, for such a comparison the same element abundances, the same number of angles (i.e., the one standard angle cosine $1/\sqrt{3}$ that we typically use in RAD2L), and the same weights given by equations (17)–(21) were selected. For MULTI, in

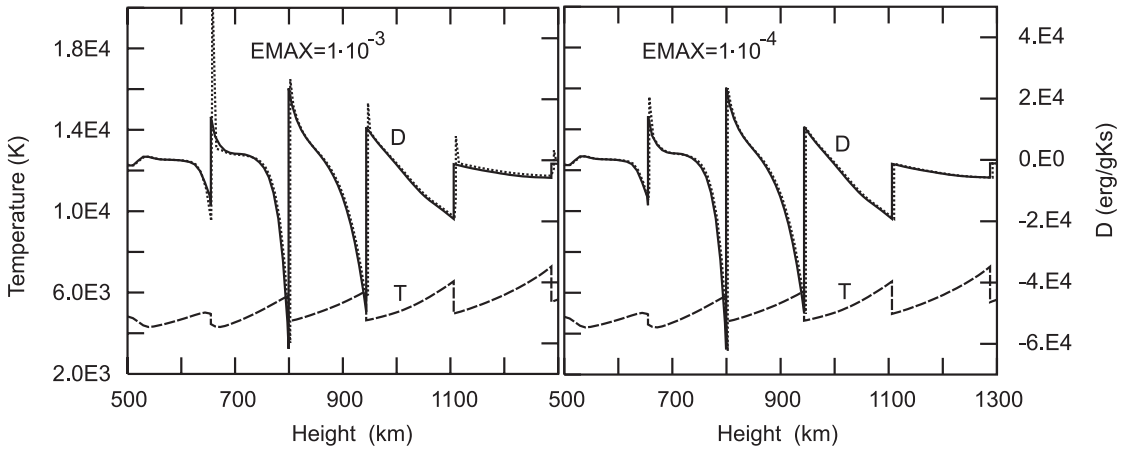

FIG. 1.—Damping functions D with RAD2L (*solid line*) and MULTI (*dotted line*) for the Ca II K line using two-level atom models plotted vs. height z together with the temperature T for a solar chromosphere. The left panel is for the MULTI parameter EMAX $= 1.0 \times 10^{-3}$, the right panel for EMAX $= 1.0 \times 10^{-4}$. The plot is for a phase at time $t = 1483.3$ s of an acoustic wave calculation with period $P = 20$ s and energy flux $F_M = 1.7 \times 10^8$ ergs cm^{-2} s^{-1}.

addition, we had to specify the parameter EMAX, the maximum allowed relative change per iteration of the number densities of all bound levels. Figure 1 shows that the accuracy of the solution critically depends on EMAX and that the error limit EMAX has to be quite small, namely, as low as EMAX $= 5.0 \times 10^{-5}$, to get reliable results in the shock regions.

To reach this accuracy in MULTI, at least 1725 iterations for Ca and 3033 for Mg in our mentioned test calculation were required. Since both RAD2L and MULTI use a diagonal operator for the solution in the operator perturbation method, it is surprising that RAD2L needed only about 20 iterations to reach an accuracy of EMAX $= 1.0 \times 10^{-6}$. This shows that the use of RAD2L with cooling rate correction factors will bring a large saving of computer time. In addition, we found that, contrary to RAD2L, the number of iterations needed for MULTI depended strongly on the number of shocks in the atmosphere and on the strength of these shocks.

A more detailed comparison for both the Ca II K and Mg II k lines for the entire height range of our wave calculation, extending from $z = 0$ to 3000 km, is shown in Figure 2. Here all 1344 height points of the four wave phases with time steps $\Delta t = P/4$ apart are shown. The solid lines represent the slope $K = 1.0$, and for MULTI we have used EMAX $= 5.0 \times 10^{-5}$. It is seen that most points lie on the line with slope $K = 1.0$, which is expected when MULTI and RAD2L give the same results for the same atmosphere. As the deviation from the $K = 1.0$ line rapidly decreases when the parameter EMAX becomes smaller, we conclude that, given a low enough EMAX, MULTI closely reproduces the RAD2L results for both the Ca II K and Mg II k lines.

Testing MULTI for a large number of other wave phases and other wave periods, we encountered another problem. Contrary to RAD2L, we found a number of cases in which MULTI did not converge. Moreover, the crash sometimes occurred only for one of the two (Ca II K or Mg II k) lines. We attribute this in part to the fact that our height grid with its regularly spaced height intervals and interspaced shock points may pose unsuitable optical depth intervals for MULTI's Newton-Raphson iteration. This problem appeared particularly when strong shocks were treated and low front shock temperatures occurred. Another situation where MULTI did not usually converge was close to and in the process of shock overtaking, when two shocks approached one another and subsequently merged.

4. WAVE CALCULATIONS TO DETERMINE THE RATE CORRECTION FACTORS

After the comparison in the two-level cases, we now compare the results of RAD2L and MULTI for a five-level Ca II (K, H,

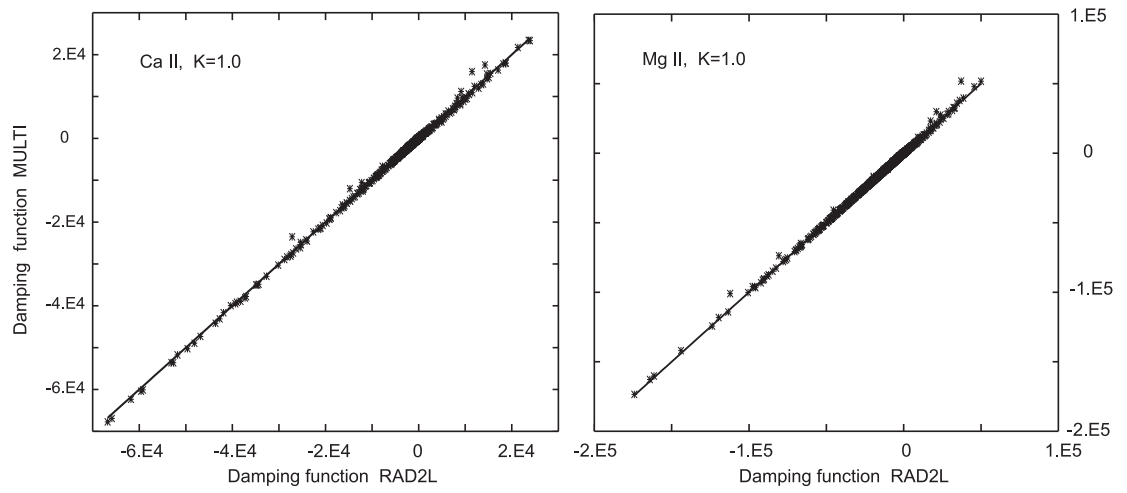

FIG. 2.—Individual comparisons for the Ca II K and Mg II k lines of all 1344 MULTI and RAD2L damping functions at identical heights of four wave phases at times $t = 1483.3$, 1487.9, 1492.8, and 1497.3 s by use of a two-level atom and parameter EMAX $= 5.0 \times 10^{-5}$ in MULTI.

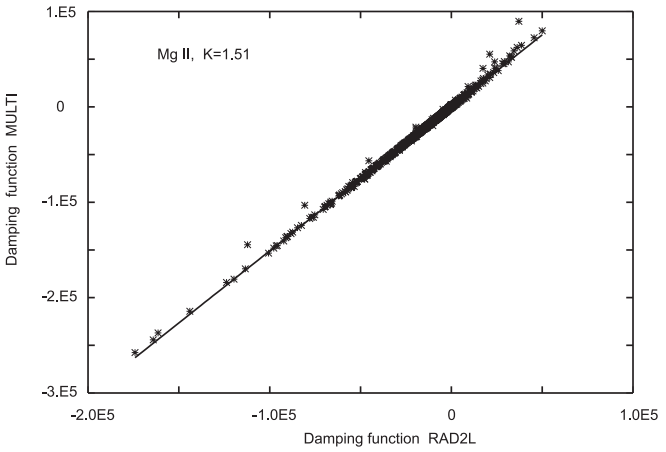

FIG. 3.—Individual comparisons for Mg II of the full MULTI and the RAD2L damping functions D for the 1344 height points of the four wave phases at times $t = 1483.3$, 1487.9, 1492.8, and 1497.3 s. This is with the MULTI parameter EMAX $= 5.0 \times 10^{-5}$. Also shown is a best-fit line with a slope of $K = 1.51$.

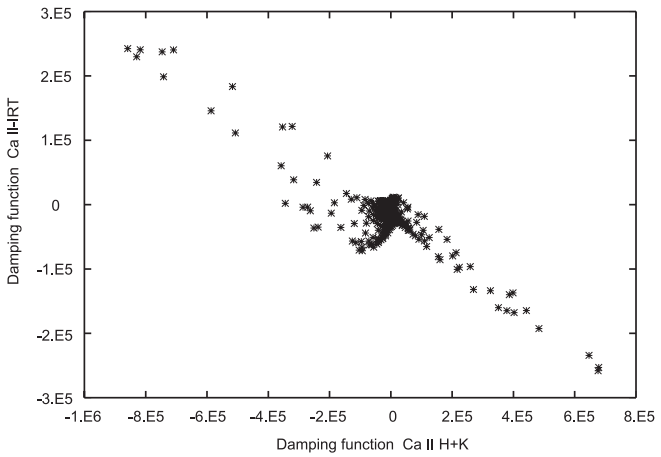

FIG. 5.—Damping functions D of the Ca II IRT emission vs. the resonance line H+K emission using MULTI for all of the 1344 height points of the four wave phases.

and IRT lines) plus continuum and a three-level Mg II (k and h lines) plus continuum atomic model. The radiative transfer in MULTI is now computed with three angle points and using an error parameter EMAX $= 5.0 \times 10^{-5}$. Figures 3 and 4 show the same four phases as in Figure 2. The solid lines in the figures describe least-squares fits to the data. Their slopes K (with $K = 1.51$ for Mg and 7.08 for Ca) directly constitute the desired cooling rate correction factors.

As seen from Figure 3, such a factor for Mg II describes rather well the total radiation loss of this ion produced by adding the second resonance line. For the more complicated Ca II ion (Fig. 4), with its IRT lines, a scaling using a cooling rate correction factor is less accurate. This is because the resonance lines Ca II H and K typically act in opposite direction compared to the IRT lines. That strong heating in the resonance lines is correlated with cooling in the IRT lines while strong cooling in the H+K lines is associated with heating in the IRT lines is shown in Figure 5. However, because the losses and gains in the IRT lines are considerably smaller than by the resonance lines, one still has a reasonably good agreement between the full MULTI and the scaled RAD2L results, as seen in Figure 4. Note that in Figures 3, 4, and 5 we have used all 1344 values of the damping functions D of the four wave phases mentioned above.

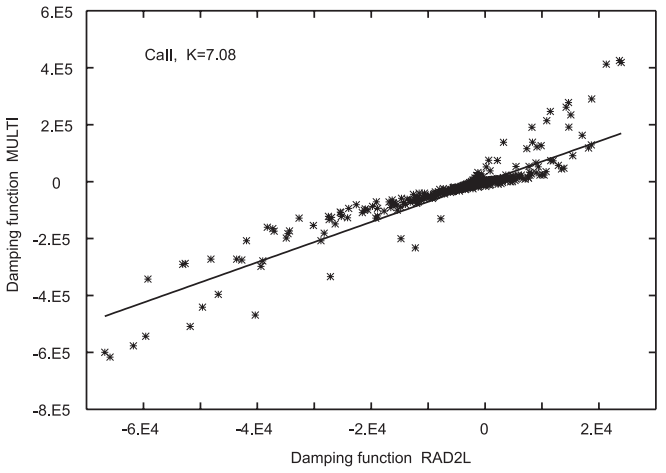

FIG. 4.—Same as Fig. 3, but for Ca II. The slope of the best-fit line is $K = 7.08$.

For the phase at $t = 1483.3$ s, Figures 6 and 7 show a detailed comparison of the MULTI and scaled RAD2L damping functions versus height. It is seen that for Mg there is an almost perfect agreement. Due to the mentioned opposing behavior of the resonance and IRT lines, the agreement for Ca II is less good. Different from the Mg II results, we notice that particularly at the shocks the damping function amplitudes with MULTI are much larger than the scaled RAD2L amplitudes. This fact, also seen in Figure 4, is explained by the presence of the IRT lines, which generally leads to higher populations of the 2P levels in Ca II. However, for Ca II, as seen in Figure 5, the resonance lines are always more important than the IRT lines.

In the cool parts directly in front of the shocks, that is, the region to the right of the shocks in Figure 7, one has radiative heating (positive D-values). Here the heating in MULTI is generally enhanced relative to RAD2L. Both MULTI and RAD2L show that the front shock regions are heated by resonance lines. But due to the opposing behavior of the IRT lines against the resonance lines, the cooling by the IRT lines in the front shock region generates a rapid decrease of the resonance line heating and thus of the total heating in MULTI as compared to the scaled

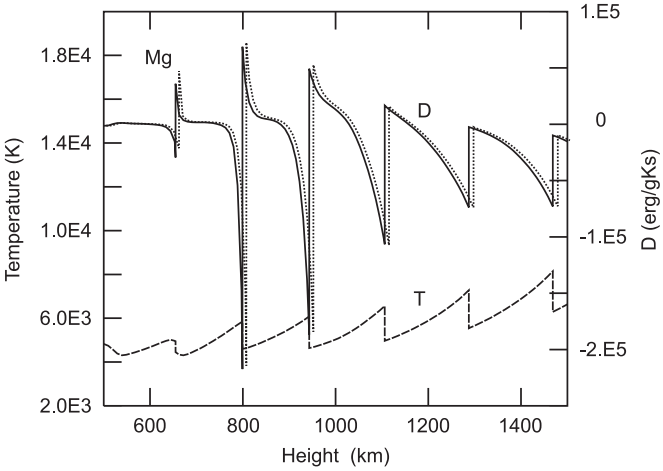

FIG. 6.—Damping functions D with RAD2L (*solid line*) and MULTI (*dotted line*) for the total Mg II emission vs. height z together with the temperature T. The MULTI results have been shifted to slightly greater height for better viewing. The values are for the acoustic wave phase of Fig. 1. The RAD2L Mg II results have now been multiplied with a cooling rate correction factor $f_{\mathrm{Mg}} = 1.51$.

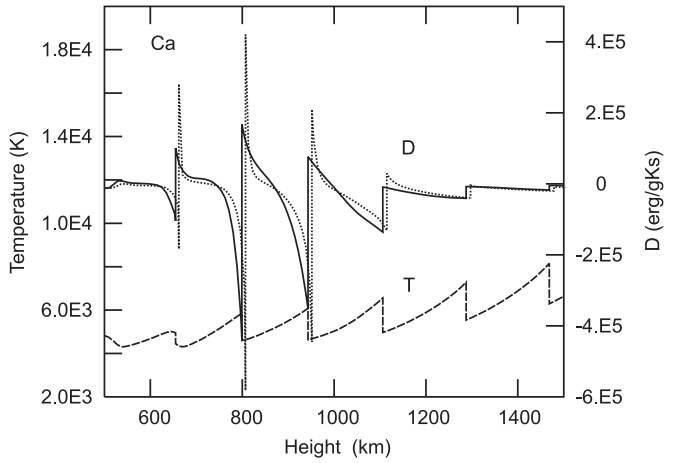

FIG. 7.—Same as Fig. 6, but for Ca II. The RAD2L Ca II results have been multiplied by a cooling rate correction factor $f_{\mathrm{Ca}} = 7.08$.

values of RAD2L. This produces a more spiky behavior of the radiative heating in MULTI than in RAD2L.

In the postshock regions to the left of the shocks, where the main radiation loss due to resonance line cooling occurs (negative D-values), the negative amplitude of the Ca II damping function in MULTI is also enhanced relative to the scaled values of D in RAD2L (Fig. 7). Here the radiative heating in the IRT lines leads to a reduced cooling in the resonance lines. Same as in the front shock region, the back shock region also acquires a more spiky behavior in MULTI.

The large differences of the MULTI and RAD2L Ca II D-values in the front shock regions can also be seen in Figure 4: points with large positive D-values (front shock regions) are not well fitted with the scaling factor 7.08. Two different scaling factors, one for the negative and one for the positive D-values, would result in a better fit. However, because there are only a few points with such large deviations (approximately a dozen points from 1344) and because the use of a best-fit function (instead of a single scaling factor) involves the risk of serious runaway points, we have confined ourselves to only one but robust scaling factor.

Until now we have concentrated on a wave with a 20 s period. Employing waves with other periods, we get a similar behavior. Excellent linear scaling between MULTI and RAD2L is obtained for the Mg II losses and reasonable linear scaling for the Ca II losses. However, while the cooling rate correction factors $f_{\mathrm{Mg\,II}}$ for Mg II are essentially independent of the wave period, we find that the Ca II rate correction factors $f_{\mathrm{Ca\,II}}$ depend on the wave period. Table 1 shows the obtained cooling rate correction factors for monochromatic wave calculations with periods of $P = 20$, 40, 60, and 75 s, as well as for a stochastic wave; the latter was represented by a linear superposition of 100 sinusoidal partial waves with amplitudes determined from the turbulent energy spectrum considered by Ulmschneider & Musielak (1998), frequencies higher than the local acoustic cutoff, and random phases. The Mg II factors vary little, while for the Ca II factors one has substantial variations. Here, apparently, the magnitude of the shock jump, which increases with the wave period like $\sim P^2$, plays an important role.

The linear correlation coefficients r are given by

$$r = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}, \qquad (24)$$

TABLE 1

COOLING RATE CORRECTION FACTORS AND LINEAR CORRELATION COEFFICIENTS

PERIOD (s) (1)	Ca II		Mg II	
	f (2)	r (3)	f (4)	r (5)
20	7.08	0.914	1.51	0.999
40	6.38	0.912	1.48	0.998
60	5.21	0.939	1.47	0.998
75	4.61	0.932	1.46	0.996
Stochastic	5.55	0.923	1.51	0.999

NOTES.—Cooling rate correction factors f and linear correlation coefficients r for scaling the Ca II K and Mg II k line losses, respectively, to the total losses, for monochromatic waves of indicated period and for stochastic waves. In all models we have used an energy flux of $F_M = 1.7 \times 10^8$ ergs cm^{-2} s^{-1}.

where x_i and y_i are the Mg/Ca D-values of MULTI and RAD2L for all i points, respectively, and $\bar{x}$ and $\bar{y}$ are the mean values of D(MULTI) and D(RAD2L) averaged over all used n points.

The comparison of the cooling rates f from MULTI and RAD2L can be found in columns (2) (for Ca II) and (4) (for Mg II) of Table 1. For Mg II, all r-values (cols. [3] and [5]) are nearly 1 and show the excellent reliability of the scaling factors $f_{\mathrm{Mg\,II}}$. The correlation coefficients for Ca II are not so high, but values always >0.9 are sufficient to establish reliable scaling factors $f_{\mathrm{Ca\,II}}$. As can be seen from Figure 4, a better correlation for Ca II would be possible by use of two correction factors: one for the regions with negative radiation damping values and one for regions with positive values. This behavior is typical for all our test runs and does not depend on the wave period. But since the occurrence of large positive damping rates is rare, we do not think that the use of a second scaling factor for Ca II is required.

The fact that for Ca II the shorter wave periods have higher correction factors can be partially explained by the fact that the importance of the IRT line cooling rates compared to that of the resonance lines of Ca II decreases with increasing wave period. The stronger the shocks, the more important are the resonance lines compared to the IRT lines, and thus the closer one gets to the behavior seen for Mg II, that is, the better a scaled RAD2L describes the situation. In the back shock region, for instance, the ratio of the IRT and H+K damping functions, D(IRT)/D(H+K), varies from 0.3 to 0.13 when the wave period increases from 20 to 75 s, respectively. It should be noted, however, that although the heating and cooling contributions of the IRT lines are always much less than those of the resonance lines, it is the presence of these lines that significantly modifies the resonance line behavior.

Finally, we have tested the stability of the obtained cooling rate correction factors. For a wave with period $P = 40$ s and $F_M = 1.7 \times 10^8$ ergs cm^{-2} s^{-1}, we performed a wave calculation using preliminary cooling rate correction factors of $f_{\mathrm{Ca\,II}} = 4.31$ and $f_{\mathrm{Mg\,II}} = 1.42$ in the radiation routine RAD2L. Using wave phases from this calculation, we subsequently performed the analysis of MULTI versus RAD2L as described above to get new correction factors (henceforth called iteration 1). Subsequently a part of the atmosphere calculation was redone with these new factors, which led to slightly different temperatures and pressures. With this new atmosphere, we calculated again new correction factors using MULTI. The results of this iterative process are shown in Table 2 for three iteration steps.

The correction factor for Mg II is very stable after the first iteration and also $f_{\mathrm{Ca\,II}}$ converges fast. Since the modification of $f_{\mathrm{Ca\,II}}$ by this iteration process is minor and because the atmospheres are nearly unaffected by small changes of the correction

TABLE 2
Cooling Rate Correction Factors for the Ca II K and Mg II k Line Losses of a Model with Wave Period $P = 40$ s for the First Three Atmosphere Iterations

Parameter	Mg II	Ca II
Start values	1.42	4.31
Iteration 1	1.48	6.38
Iteration 2	1.48	6.62
Iteration 3	1.48	6.70

factors, we feel that it is not necessary to do more than the first iteration for a typical solar wave calculation. From this we conclude that the derived correction factors are quite stable.

5. CONCLUSIONS AND DISCUSSIONS

We have shown that a fast and reasonably accurate method for calculating the total radiative losses in the Ca II H, K, and IRT lines and in the Mg II h and k lines can be developed. The method called RAD2L is based on a two-level atom and a pseudo–partial frequency redistribution (pseudo-PRD) procedure, and it is efficient in the computation of theoretical models of stellar chromospheres because only the single K and k lines are calculated. It can be demonstrated that the total radiative losses in the Ca II and Mg II lines are reproduced when the single K and k line losses are multiplied by cooling rate correction factors. These correction factors are 1.5 for Mg II and between 5 and 7 for Ca II, depending on the wave period (which range from 20 to 75 s, respectively).

For Mg II the total radiation due to the added resonance line is described very well by RAD2L using the correction factor 1.5. For Ca II the radiation loss and gain are still dominated by the resonance lines, but the radiation is considerably enhanced due to the presence of the IRT lines. These IRT lines show an opposite radiation behavior than that of the resonance lines. This opposing behavior leads to a more spiky dependence of the radiation near shocks when using MULTI compared to RAD2L.

The presented radiation code RAD2L allows us to obtain a one-dimensional chromosphere model in radiative equilibrium with a few hours of computation time on a PC. The necessary cooling rate correction factors, which depend on the wave period, can either be preselected or quickly evaluated using MULTI on a few wave phases. This allows us to rapidly produce chromosphere models with realistic cooling rates. Moreover, using MULTI at certain phases in a wave code, or after the wave computation is completed, one can evaluate and easily correct the error in the total radiation losses.

The procedure in RAD2L is fast because the radiation treatment is confined to two bound level atom models. Moreover, the employed operator perturbation method converges very rapidly, regardless of the hydrodynamic situation (narrow optical grid distances, strong or merging shocks). Convergence problems were never encountered. Compared to a hydrodynamic calculation with the full version of MULTI and a full PRD treatment, we estimate a saving of computer time by a factor of at least 10^6. However, all present wave calculations use pseudo-PRD, which speeds up the computation by factors of around 10^3–10^4. We estimate that the remaining speed acceleration by using RAD2L instead of MULTI is around factors of 10^2–10^3.

In this paper, the cooling rate correction factors are calculated only for the Mg II and Ca II lines, which are among the main sources of radiative losses in stellar chromospheres. Studies by Anderson & Athay (1989) showed that other lines, especially by Fe II (see also Carlsson & Stein 1997, 2002), can also be important sources of chromospheric radiative losses. In general, it should be possible to extend the method to Fe II and other elements, and these topics will be the subject of investigation in a forthcoming paper.

A disadvantage of the described method is that the radiation losses in complicated cases like Ca II cannot be computed as accurately as in MULTI and that one has the principal need to establish correction factors for every new model. A change of parameters like effective temperature, gravity, metallicity, mechanical energy flux, or wave period may also change the correction factors. However, in such cases it seems reasonable to interpolate the factors from a few corner values like those shown in Table 1. We plan to extend our investigations from the Sun to other stars with the aim to develop correction factor tables (or interpolation formulae) for various elements, depending on the stellar parameters.

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